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A Review of Logic for Computer Professionals

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SCHOOL OF COMPUTER SCIENCE AND INFORMATION SYSTEMS

TECHNICAL REPORT

Number 204, August 2004

A Review of Logic for Computer Professionals

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PACE UNIVERSITY

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I like to hike the Westchester North County Trailway, snorkel, and read science fiction - not all at the same time.

A Review Of Logic For Computer Professionals

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ABSTRACT

Logic is used in programming and circuit design. I introduce the basic definition of Logic and give some concrete examples.

By "Logic" I mean the representation and manipulation of variables that can be in one of two states. In programming the variables stand for conditional statements used in looping, that can be true or false. In circuit design the variables stand for voltage on or off or current flowing or not flowing.

A section exhibits some applications of Logic to computer language "if" and "while" statements.

The appendix shows an example of XOR applied to RAID storage, the proof of the sufficiency of Sheffer Stroke (NAND), and some examples from formal circuit theory. They are in the appendix because they require mathematical maturity (and some motivation) to fully follow them.

A bibliography points to standard texts for more detailed information.

This paper is not meant to substitute for the bibliographic references. It is meant as a quick and broad introduction, and as a short pertinent reference for classes using these topics.

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I. Preliminary Ideas And Terminology

A). Two-state Variables (TSV)

It's always easier to think in terms of canonical examples. So, instead of being abstract and talking of two-state variables, we will think of variables that stand for statements that are true or false. Later, we can think of 0 voltage as false and high voltage as true (or vice versa) and apply our new knowledge to circuits.

If I state that P is a statement that is true or false, I am being half abstract since I am not giving an actual statement that is true or false. We can think of a canonical example: "I worked more than 40 hours this week." This is a statement. It is either true or false.

B). Functions Of Two-state Variables (TSVF)

There are a bunch of useful functions of <u>two</u> two-state variables that we regularly use in computing. If P is a TSV and Q is too (ha ha), then we can represent their possible separate values as P=0 or P=1 and Q=0 or Q=1.

Variables that can take on only two values are called Boolean variables. In fact this whole topic of formalization of classical logic was started by George Boole in the century before last.

If P stands for "I worked more than 40 hours this week" and Q stands for (represents) "I got paid for only 40 hours work", then the two variable function

P AND Q

represents the TSVF "I worked more than 40 hours this week and I got paid for only 40 hours work." This compound statement is also a TSVF since it is either true or false.

WEIIII .... Maybe.

We have to agree exactly on what is meant by the language conjunction "and".

There has been an interaction between language and Logic so that today we have very precise meanings for words like AND, OR, and IMPLIES, all of which are two Two-state Variable Functions. The meanings are the ones defined in Logic.

C). Truth Table Representation Of Functions Of Two-state Variables

A Truth Table is a standard form display of the input variables (TSVs) and the output of the function (a TSV). The function is symbolized and put in a header – see below for examples of AND, OR, NOT, IMPLIES, TRUE, and FALSE.

AND, OR, IMPLIES, TRUE, and FALSE are two-variable functions. TRUE is the TVF that gives the value 1 (true), no matter what the input variable values are.

Similarly, FALSE is the TVF that gives the value 0 (false), no matter what the input variable values are.

NOT, however, is a one-variable function.

The symbols for the functions are fairly common ones coming from the Latin word for the functions. $\wedge$ means AND, and $\vee$ symbolizes (means) OR.

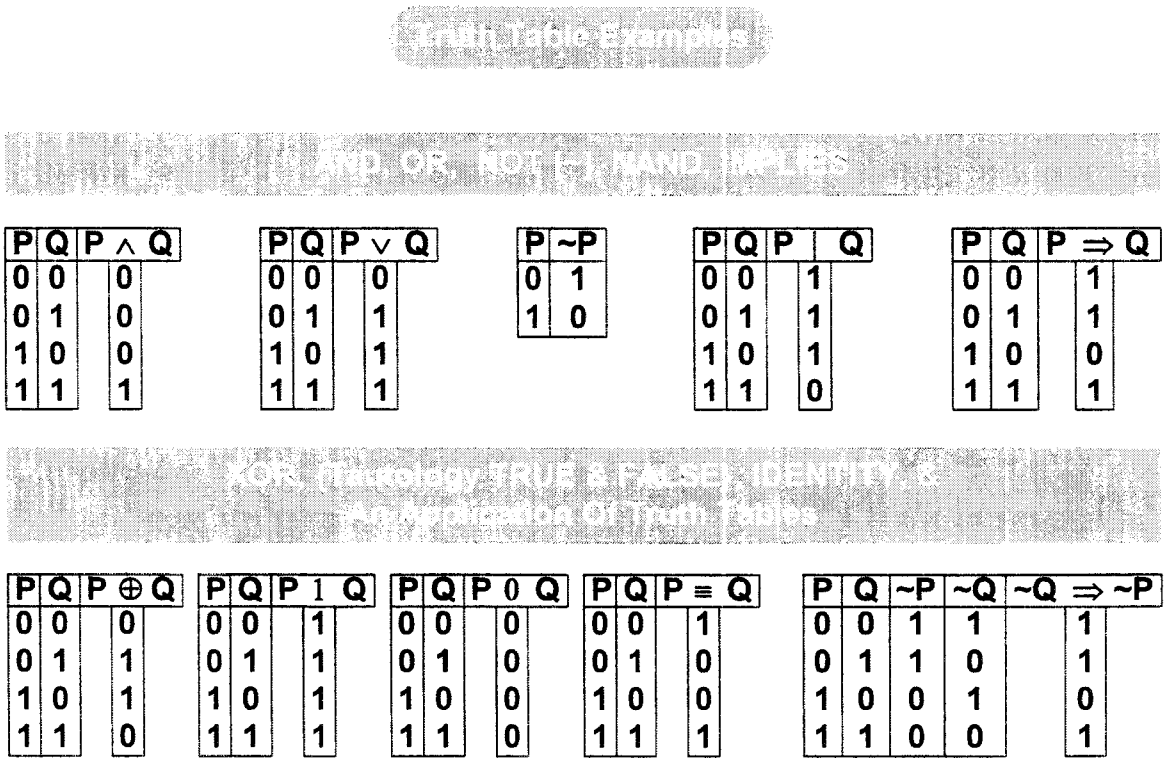
NAND (symbolized by the Sheffer "Stroke", or stile character) is used in circuits. There is also a NOR function.

IMPLIES is meant to capture some of the meaning of implication in language. "All humans are mortal AND I am a human IMPLIES I am mortal". In logic, we take IMPLIES to mean something a little stronger the usual use of implies in language. If you look at the truth table for $\Rightarrow$, which is the symbol for implies, you see that IMPLIES is false ONLY when the premise is TRUE and the conclusion is FALSE. The implication (ho ho) of this is that, from the table, we see FALSE IMPLIES anything. Assume a falsehood, and I you can validly imply anything I want. The general format of a Truth Table is:

Input 1	Input 2	Output Function
Value 1	Value 1	Output 1
Value 2	Value 2	Output 2
Value 3	Value 3	Output 3
Value 4	Value 4	Output 4

Where all input values are TRUE or FALSE and each output is TRUE or FALSE

Figure 1: Truth Table Examples Of Basic Two Two-state Variable Functions



Notice NAND is the NOT of AND. NOR is the NOT of OR, [work it out or see Figure 2 below]. The last application example shows how to add helper columns to ease table construction.

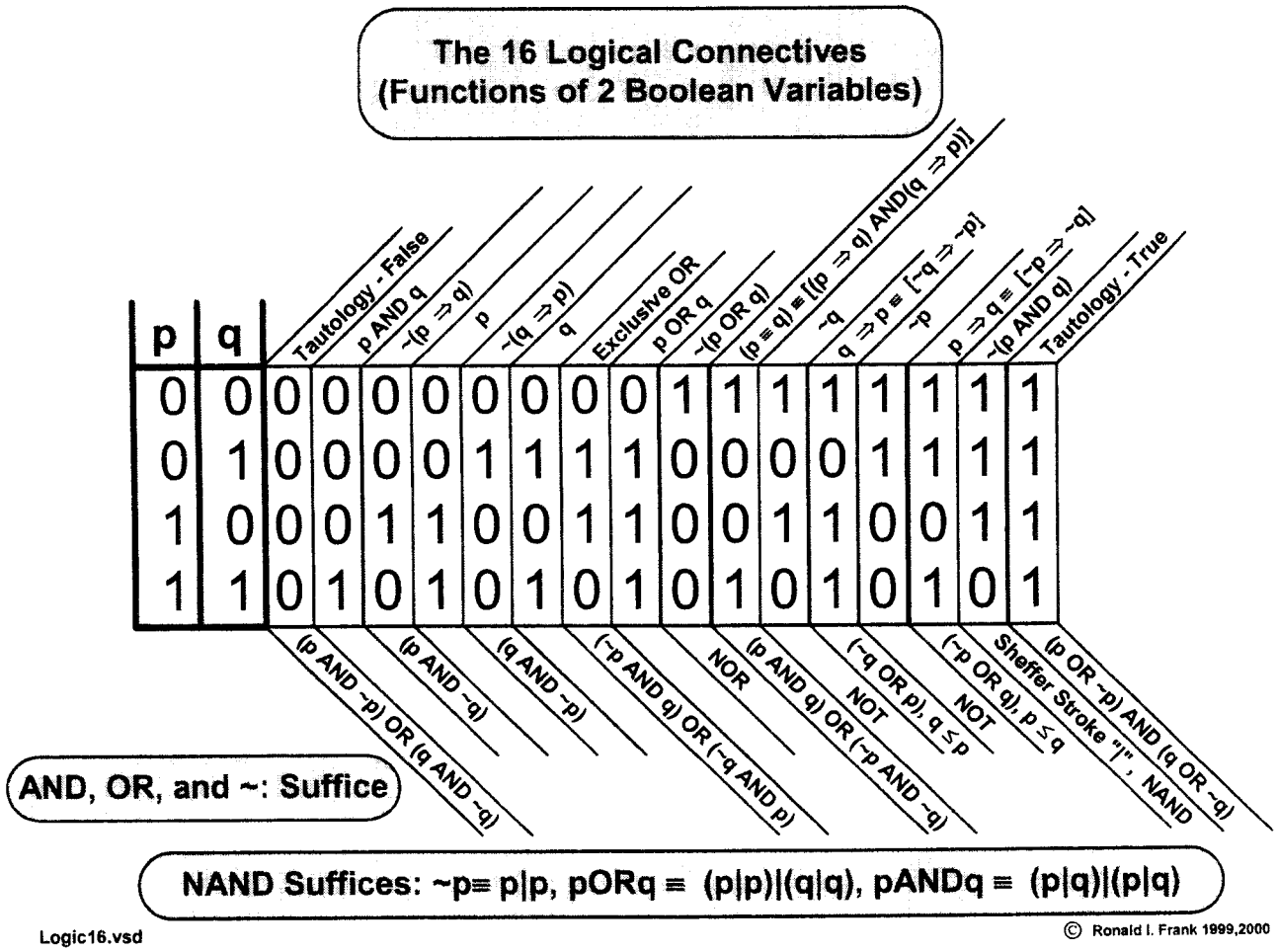
Notice that the output of a two variable function is itself a TSV in that only the values 1 and 0 appear. Since there are 4 input cases [0 0, 0 1, 1 0, 1 1] there are 4 possible output cases of 0 or 1. This means that there are 16 possible two variable functions [2x2x2x2=16 possible outcomes].

I present for your reference, the 16 cases and their symbolic interpretation.

NOTE: Every one of the 16 possible two variable functions can be represented using only the functions AND, OR , and the NOT. We say that AND, OR, and NOT suffice to generate all the others.

Similarly the Sheffer stroke alone(!) can generate all of the others! The implication of this is that a single circuit element can build all logic circuits! [The same can be said of the logic function NOR]/

Figure 2: The 16 Logical Functions of Two Boolean Variables



Notice the groupings of the columns [# of Columns = total of 16]:

1. [1] NOR is NOT of OR.
2. [1] NAND is the NOT of AND.

3. [2] p output reproduces the TRUE inputs of p. NOT p reproduces the FALSEs.
4. [2] q output reproduces the TRUE inputs of q. NOT p reproduces the FALSEs.

5. [1] Output TRUE is TRUE for all possible inputs.
6. [1] Output FALSE is FALSE for all possible inputs.

7. [2] AND and OR (and the NOTs) have no further breakdown into equivalents.

8. [1] IMPLICATION ($p \Rightarrow q$) has equivalents and can be defined as $p \leq q$.
9. [1] CONTRAPOSITIVE ($q \Rightarrow p$) has equivalents and can be defined as $q \leq p$.
10. [2] The NOT of IMPLICATION AND CONTRAPOSITIVE have equivalents.

11. [1] IDENTITY ($\equiv$) is when the inputs are equal.

12. [1] EXCLUSIVE OR ($\oplus$) is often used in terms of its equivalent expressions.

II. Logical Functions As Circuit

A). [AND, OR, NOT]

If we think of a simple lever switch that can make or break a contact, we can represent some of the logical functions as simple circuits with switches in them. Some of the functions require double throw switches – that can make or break on either a left throw or right throw of the switch.

Figure 3: Series Parallel Circuits [AND, OR, NOT]

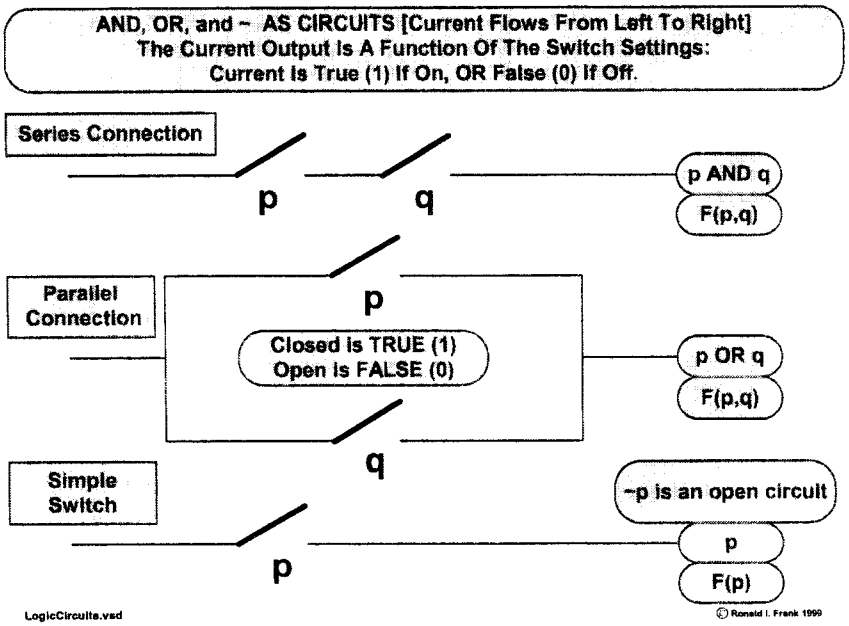
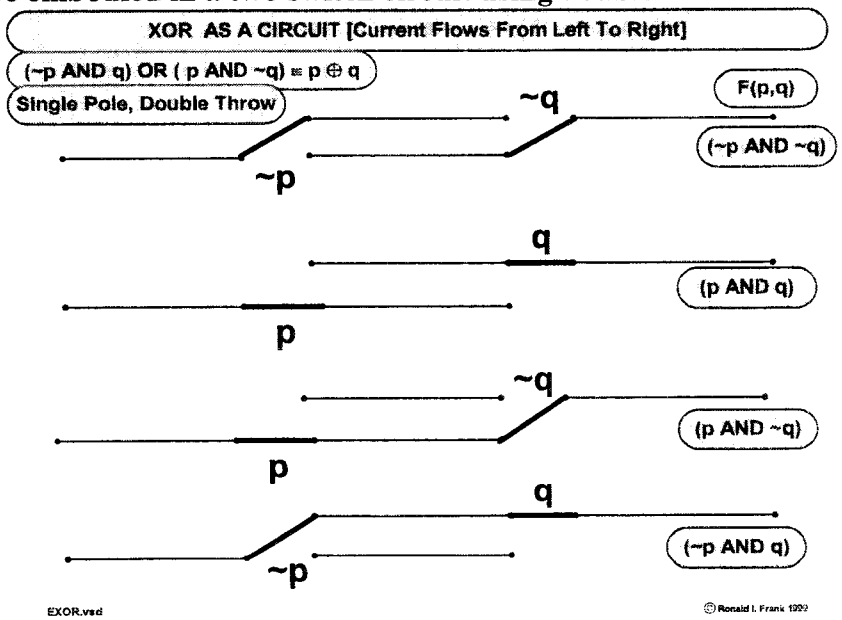


Figure 4: XOR as a circuit

B). XOR As A Circuit

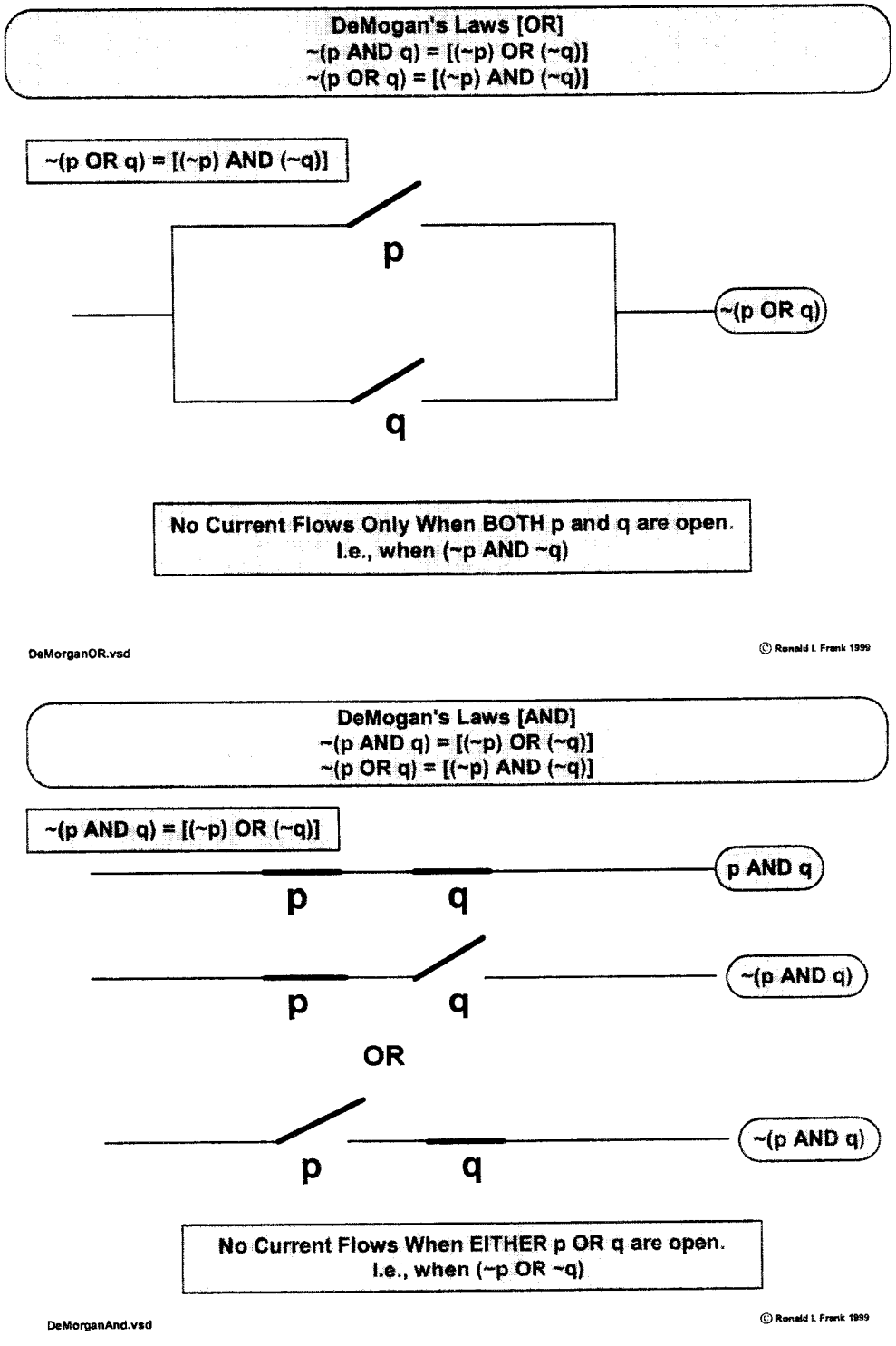
Exclusive or can be embodied in a two switch circuit using double throw switches.



C). DeMorgan's Laws As Circuits

There are a pair of very useful relationships, or laws, used to convert the negations of ANDs to ORs and vice versa.

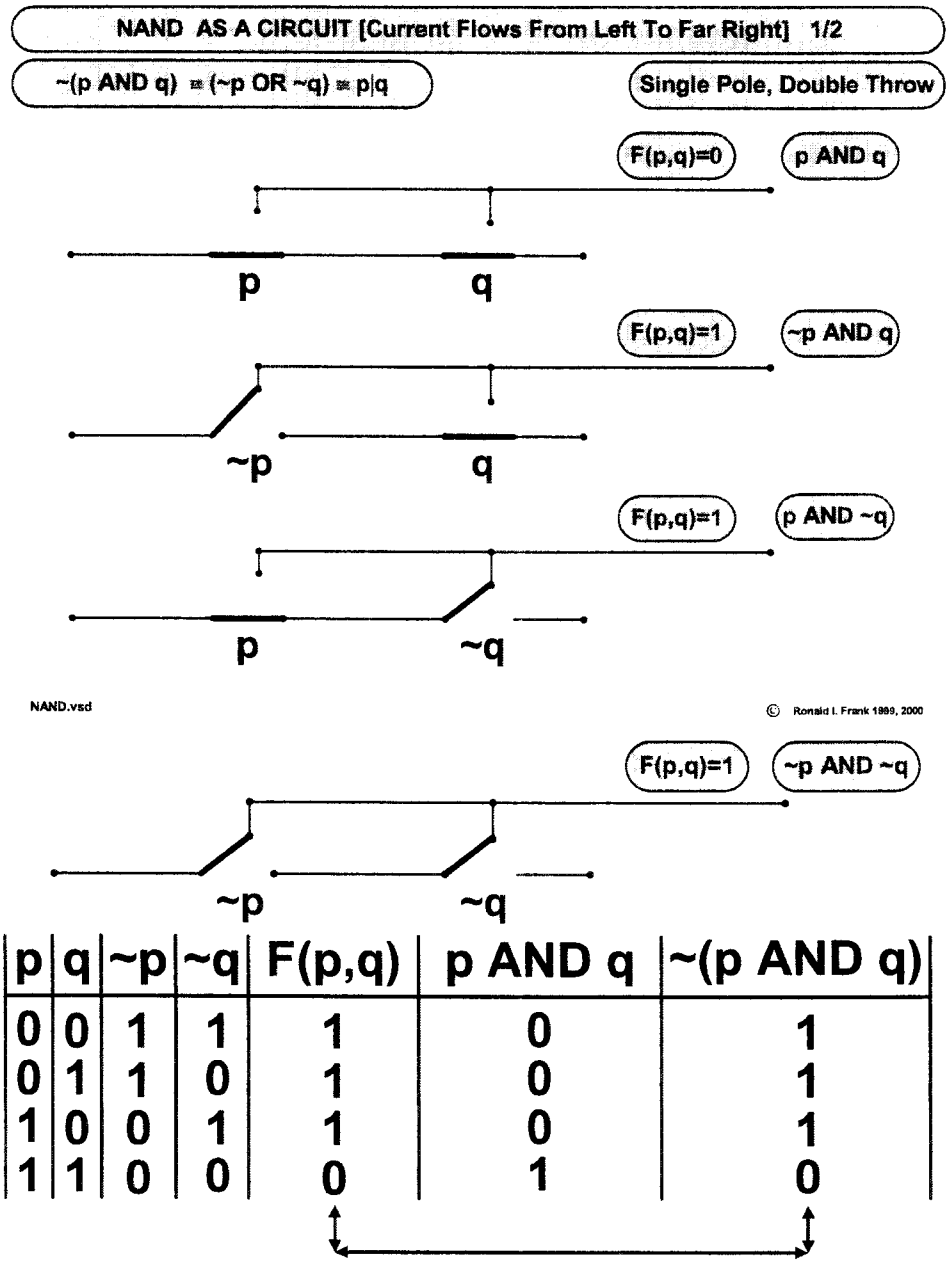
Figure 5: DeMorgan's Laws As Circuits



D). NAND As A Circuit

NOT AND can also be embodied in a two switch circuit using double throw switches.

Figure 6: NAND As A Circuit



p	q	~p	~q	F(p,q)	p AND q	~(p AND q)
0	0	1	1	1	0	1
0	1	1	0	1	0	1
1	0	0	1	1	0	1
1	1	0	0	0	1	0

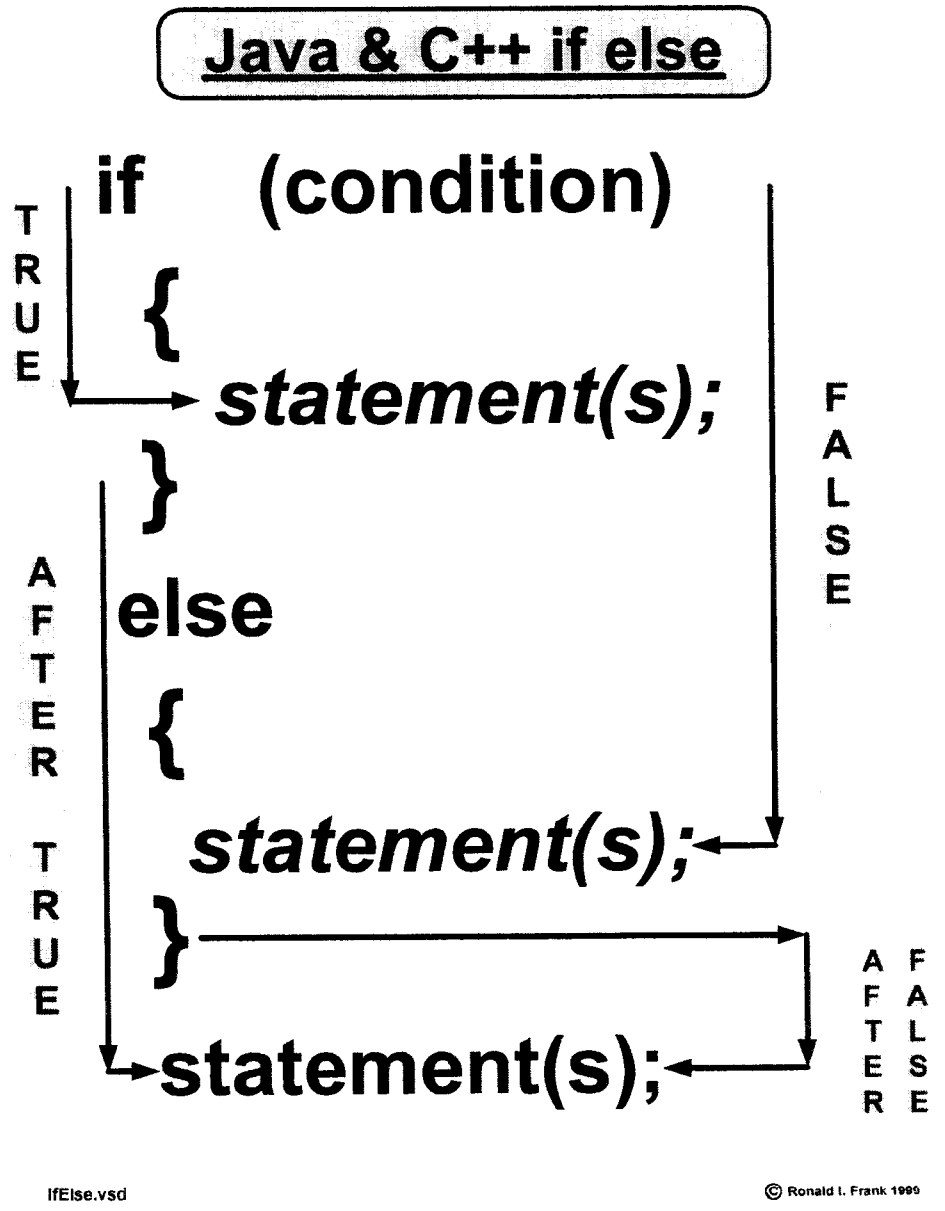
III. Application Of Logic To Programming Language Conditionals

A). [if]

The Java and C, C++ if-statement depends on a conditional (a statement that is TRUE or FALSE).

The languages allows the use of AND (&&), OR (||), and NOT (!) in the statement of the conditional. As we have seen above, this means any of the logical functions can be programmed into the conditional, including exclusive OR.

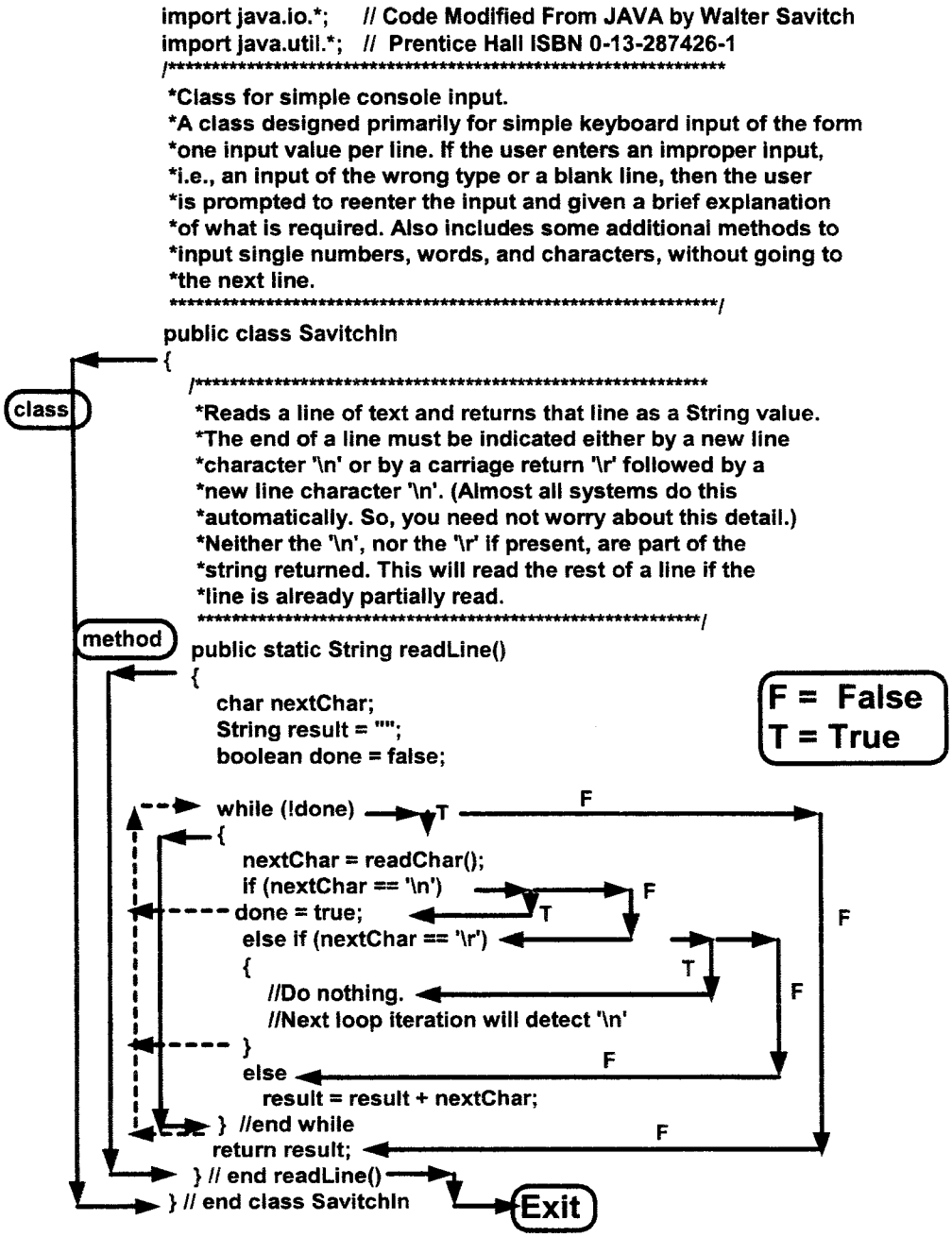
Figure 7: Tracing The Simple if-else Conditional Statement



B). [while]

The Java and C, C++ while-statement depends on a conditional (a statement that is TRUE or FALSE). The languages allows the use of AND (&&), OR (||), and NOT (!) in the statement of the conditional. As we have seen above, this means any of the logical functions can be programmed into the conditional, including exclusive OR. This is taken out of a text book as indicated in its comments. I notated the flow.

Figure 8: Tracing The Simple while Conditional Statement.



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IV. Appendices

A). Some Laws Of XOR [⊕] And Its Application To RAID Storage

1) DEFINITION OF "Exclusive OR", Some Of Its Properties

Truth Table

P	*Q*	*P* ⊕ *Q*
0	0	0
0	1	1
1	0	1
1	1	0

Properties

[⊕ Can be thought of as binary addition without carry]
[A and B are equal length bit strings, 0 is all 0s, 1 is all 1s]

1) $A \oplus B = B \oplus A$ — Commutivity

2) $A \oplus B \oplus C = A \oplus (B \oplus C) = (A \oplus B) \oplus C$ — Associativity

3) *Not so* $A \oplus (B \wedge C) = (A \oplus B) \wedge (A \oplus C)$ — Distributivity Over ∧

4) *Not so* $A \oplus (B \vee C) = (A \oplus B) \vee (A \oplus C)$ — Distributivity Over ∨

5) $\sim(A \oplus B) = \sim A \oplus B = A \oplus \sim B$ — Negation

6) $A \oplus 0 = A$ $\quad A \oplus 1 = \sim A$ — "Identities"

7) $A \oplus A = 0$ — Self-"Inverse"

8) $A \oplus \sim A = 1$ — The Universe

9) $A \oplus B = (A \wedge \sim B) \vee (\sim A \wedge B)$ — Definition in terms of ∨, ∧, ~

---

2) Practice Manipulating ⊕:

Theorem: **For Any A and B, equal length bit strings**

$A \leftarrow A \oplus B$
$B \leftarrow A \oplus B$
$A \leftarrow A \oplus B$

Interchanges A and B

Proof:
$C \leftarrow A \oplus B$
$B \leftarrow D \leftarrow C \oplus B = (A \oplus B) \oplus B = A \oplus (B \oplus B) = A \oplus (0) = A$
$A \leftarrow C \oplus D = (A \oplus B) \oplus A = (A \oplus A) \oplus B = (0) \oplus B = B$

---

3) Application to RAID Storage

[Redundant Array Of Inexpensive (or Independent) Disks]

We are not told the manufacturers' algorithms for striping and computing parity. We are only told that they use parity. Here is a view of how that might be done. Assume one block [A] is on Disk 1, another block [B] is on Disk 2 and their byte-by-byte parity [P] is on Disk 3. This takes 1/2 the extra space of mirroring

Example:
A = [10010111]
B = [11100010]
P = [01110101] even parity.
$P = A \oplus B$

If A is lost
$P \oplus B = (A \oplus B) \oplus B = A \oplus (B \oplus B) = A \oplus (0) = A$

If B is lost
$P \oplus A = (A \oplus B) \oplus A = B \oplus (A \oplus A) = B \oplus (0) = B$

If P is lost
$P = A \oplus B$

Example:
A = [10010111]
B = [11100010]
Q = [10001010] odd parity.
Let P=~Q and proceed as above.

B). Proof Of The Sufficiency Of Sheffer Stroke

(To Generate All 16 Boolean Functions Of Two Boolean Variables)

Proof Of Sheffer Stroke (NAND) Sufficiency

NAND Suffices: ~p ≡ p|p, pANDq ≡ (p|q)|(p|q), pORq ≡ (p|p)|(q|q)

1) ~p ≡ ~(p AND p) ≡ p|p **By Definition**

2) (p AND q) ≡ ~ ~ (p AND q) ≡ ~(p|q) ≡ (p|q)|(p|q) **By 1) Above**

3) (p OR q) ≡ [~(~p)] OR [~(~q)] ≡ ~[(~p) AND (~q)] ≡ (~p)|(~q) ≡ (p|p)|(q|q)
By DeMorgan and 1) Above

C). Formal Circuit Elements

I present the symbols for a few logic (boolean) functions [AND, OR, XOR, and NOT].

Next is the diagram for all four cases of a half adder (one bit position sum plus carry out, but no carry in). This is NOT the only possible design for a half adder.

Below this is two of the eight cases of a full adder (one bit position sum plus carry out, and a carry in). There are eight cases because there are four possible values for the two addends [00, 01, 10, 11], and two possible values for the carry in [0 or 1] – giving eight cases.

There is a sheet of four blank full adder diagrams for the reader to practice on.
Copy the diagram sheet and do all eight cases. Good luck.

What is missing here (because I think it is too large a deviation from the simple purpose of this paper) is the derivation of the designs of the half and full adders from first principles (e.g., by using a Karnaugh map)

Figure 9: Circuit Element Symbols; NOT, OR, & AND

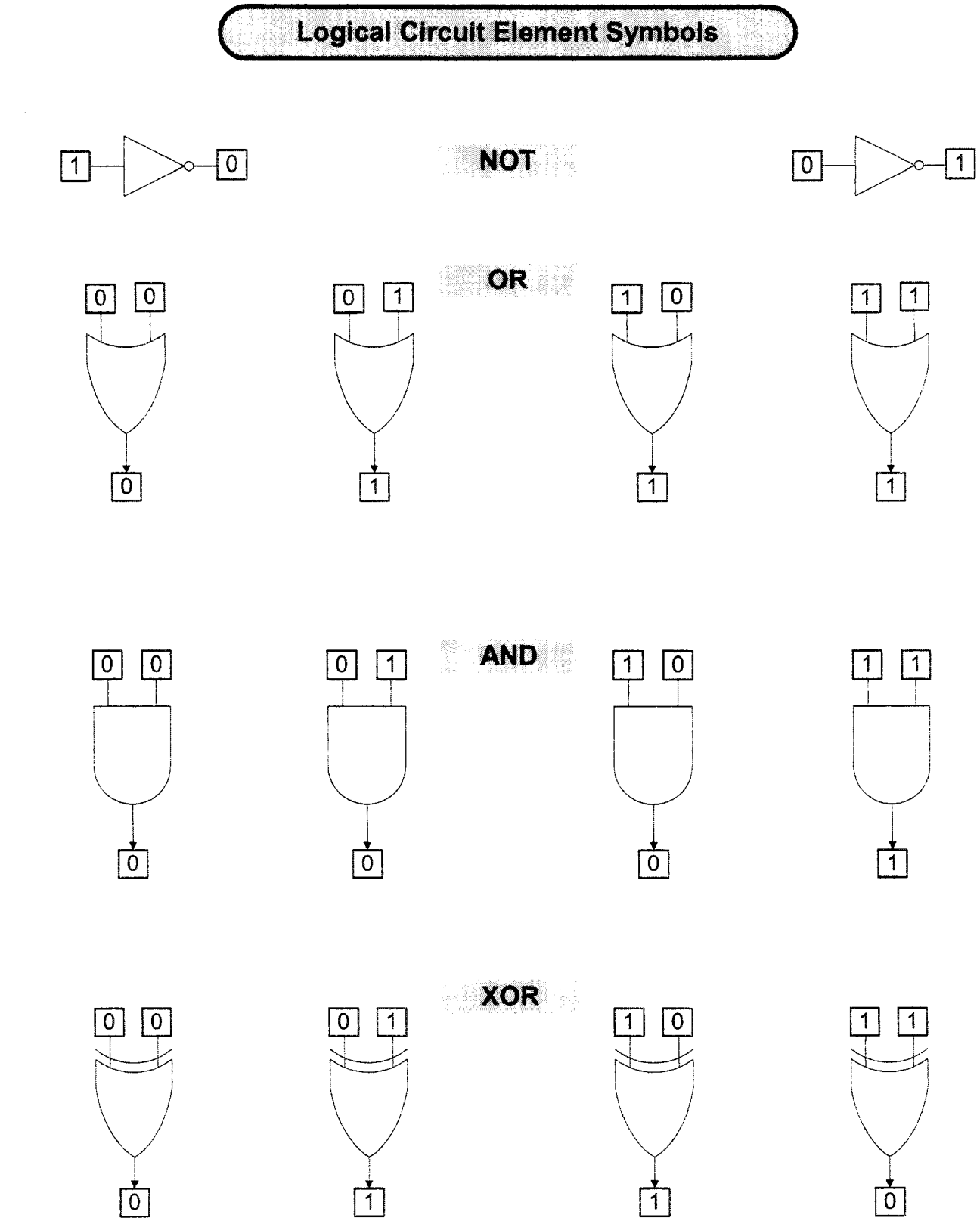


Figure 10: Half Adder Cases & Some Full Adder Cases

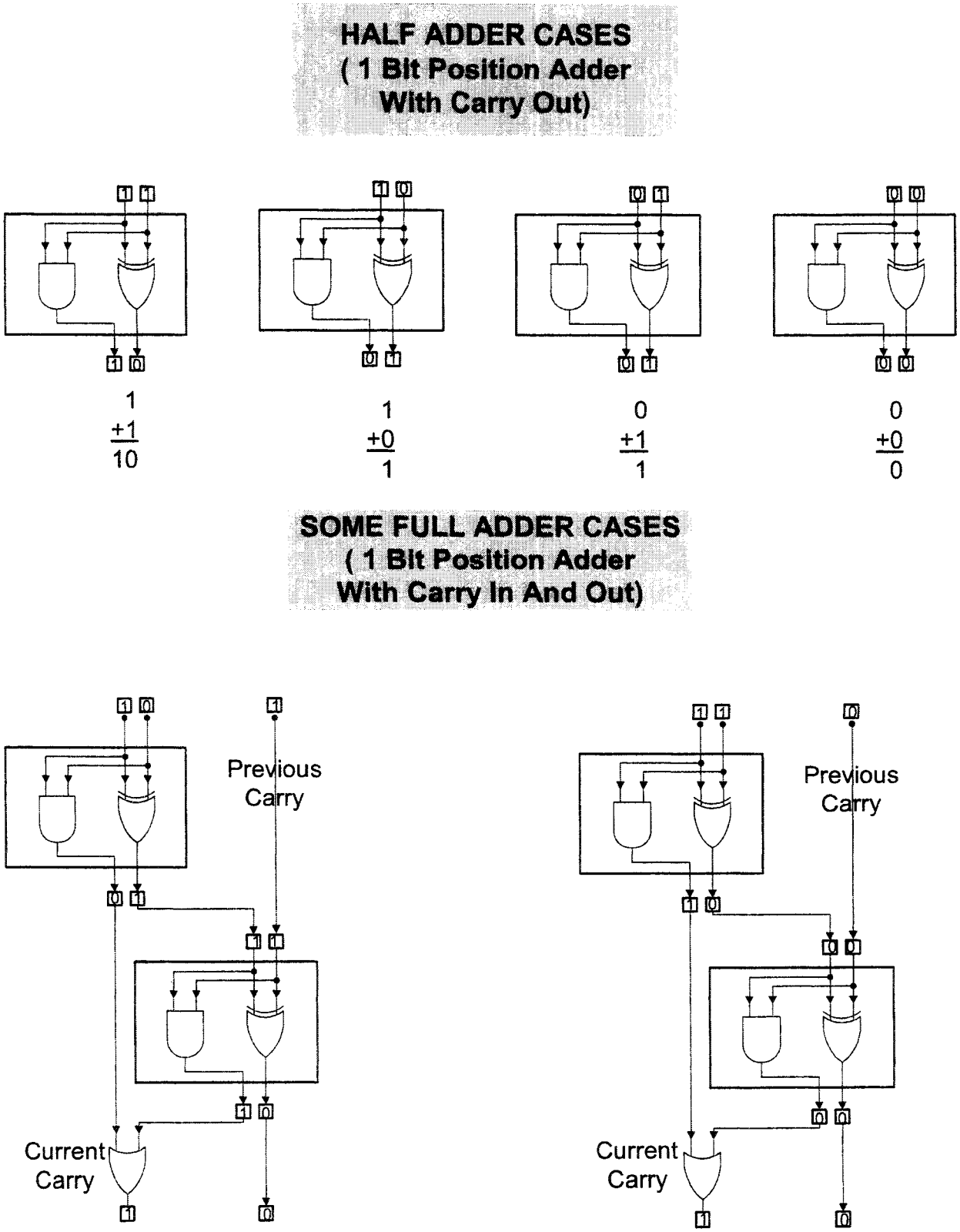
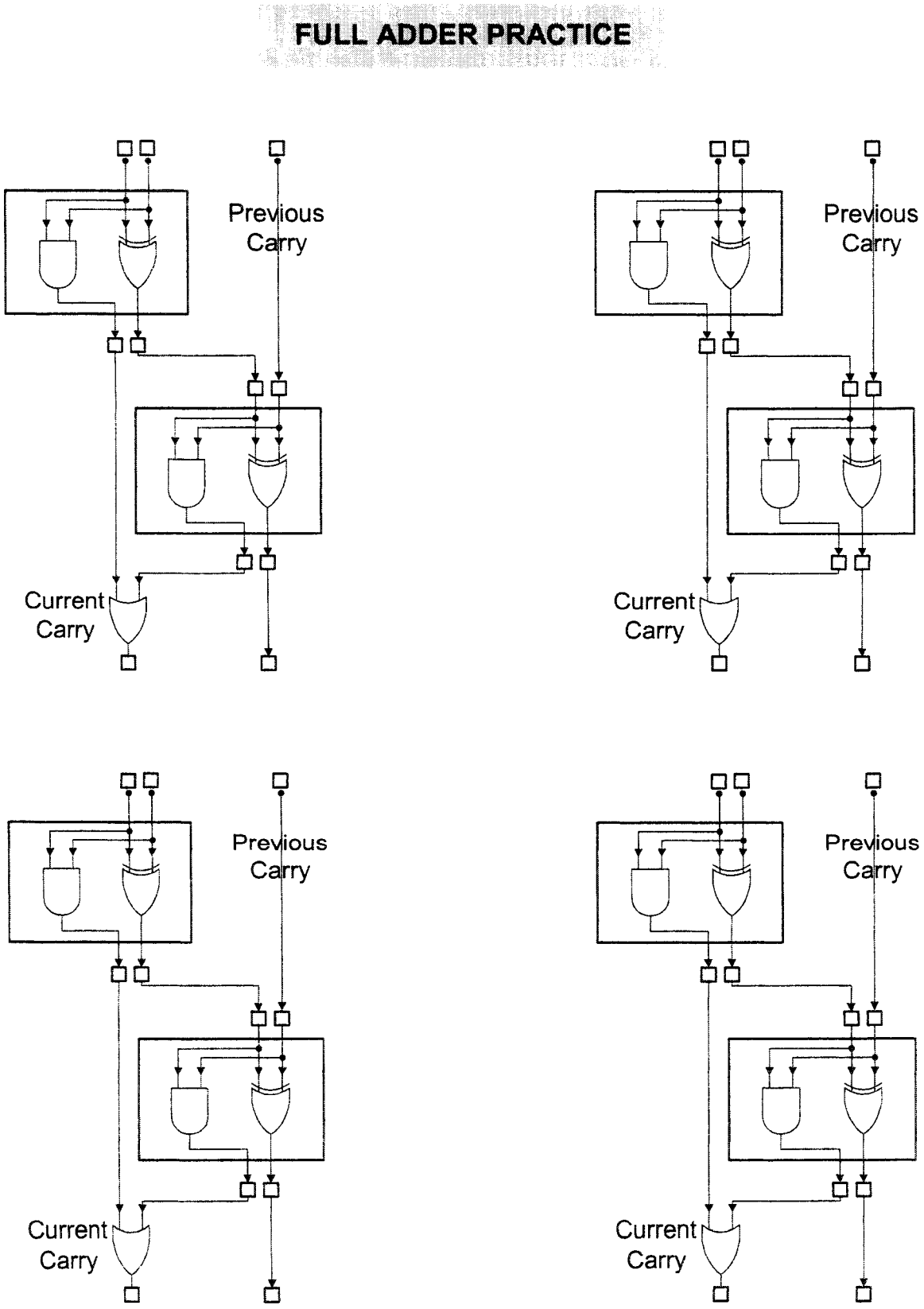


Figure 11: Full Adder Practice



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Especially pp. 608-650 on Boolean Algebra (and circuit applications)

[2] **Mano, M. M., "Computer System Architecture", 3rd Ed., Prentice Hall (1993) ISBN 0-13-175563-3**

Especially pp. 4-11 on Logic Gates and Boolean Algebra

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