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The Importance of Early Number Concepts for Learning Mathematics in Deaf and Hard of Hearing Children

Clifton Langdon

Christopher Kurz

Marie Coppola

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Clifton Langdon, Christopher Kurz, & Marie Coppola

Abstract

This chapter discusses important background knowledge and research findings from a variety of disciplines that inform best practices for supporting optimal mathematical achievement in all children. First, discussion will begin with the importance of early numeracy for later academic outcomes, and why prioritization of instruction time and early intervention are needed to increase the likelihood of a strong foundation in numeracy. Second, a brief overview of numeracy development milestones will provide a basis for discussion of our central thesis: language experiences can impact numerical cognition, which then have a significant impact on academic outcomes. Third, given the importance of numeracy skills in academic outcomes, we describe pedagogical trends that are likely to support the development of numerical cognition. This discussion will justify language remediation, increased mathematics talk, and visual-spatial representation as key goals for early intervention programs. Finally, we offer some future directions of research that will further account for underlying mechanisms of numeracy development in very young and preschool-aged children.

Keywords: *number concepts, early language, numeracy, mathematics, deaf and hard-of-hearing, pedagogy*

Introduction

Amos G. Draper, the first Deaf mathematics professor at the National Deaf-Mute College (now Gallaudet University), marveled at children's natural curiosity:

> Children, with eyes and ears opened, are filled with admiration by regularity of outline, beauty of color, and harmony of sound…As children grow to [adulthood], the love of form, color, and harmony remains central…Does it not flow from the instinctive but unrecognized perception of mathematical principles? (Draper, 1876)

Draper discussed how a child would learn mathematical principles indirectly and directly and how the knowledge would further propel the child's curiosity about the universe (Kurz, 2006, 2008). In essence, every child is a mathematician if provided with unstructured and structured, irregular and regular activities at home, in school and in the community. Draper's speech, "The Influence of Mathematical Studies upon Personal Character," is now 144 years years old. In this time, what have we learned about early childhood education for mathematics in deaf and hard-of-hearing children?

Numeracy Skills and Academic Outcomes

Various measures of children's academic readiness upon school entry and their predictive value of later academic outcomes have received much attention in the educational sciences, with considerable focus on executive function, language, and reading (Bull et al., 2011; M. L. Hall et al., 2019; Henner et al., 2016; Hrastinski & Wilbur, 2016; Mayberry et al., 2011). However, a meta-analysis of six large longitudinal studies assessing a range of cognitive, academic, and social background measures of approximately 52,000 children found that children's mathematical cognition skills at the beginning of schooling was the strongest predictor for their later academic outcomes (in both reading and mathematics) in later primary school grades

(Duncan et al., 2008). Despite the high impact of mathematical cognition on academic outcomes, the Organisation for Economic Co-operation and Development (OECD) finds approximately one-third of the United States adult population is limited to reading numbers and performing limited, one-step arithmetic operations (OECD levels 1 and below) (OECD, 2013a). The upper third (OECD levels 3-5) have a stronger sense of mathematical relationships (e.g. percentages) and can select optimal problem-solving formulae to interpret options (e.g. comparing health insurance plans). As a result, approximately 73 million Americans are not well equipped to make informed decisions about their finances (e.g., cumulative costs of loans) or health (e.g., understanding the probability of infection or likelihood of recovery from medical treatment), which has dire implications for education policy decisions and illustrate the importance of allocating valuable resources towards strong numeracy skills (OECD, 2013b).

Deaf and hard-of-hearing (d/hh) children, who have higher incidence rates of reduced language input and fluency, have documented delays relative to typically hearing children in a variety of areas of mathematical reasoning (Kritzer, 2009; Nunes & Moreno, 2002), such as counting (Nunes & Moreno, 1998), word problems (M. Hyde et al., 2003), fractions (Titus, 1995), and arithmetic comparison problems (Kelly & Mousley, 2001). These delays are well documented in Gottardis et al. (2011) which presents a meta-analysis of 23 studies comparing d/hh children and typically hearing children. However, Secada (1984) demonstrates that comparable development of number concepts is observed when comparing deaf children learning ASL from birth and typically hearing children learning spoken English when both groups have similar rote counting skills (i.e., they are able to recite the number words in order without understanding their quantity meaning). This chapter further explores the important role played by a strong language foundation for fostering numeracy skills in d/hh children.

While objective assessments of numerical skills are an important measure, mathematics anxiety and subjective self-evaluation of numerical skills also represent an important dimension in both

children (Ganley & Lubienski, 2016) and adults (Peters et al., 2019). Peters et al. (2019) compare adults who are categorized by their subjective confidence in their mathematics skills (high and low, assessed by self-report) and objectively-assessed numerical skills (high and low) on the accuracy of hypothetical medical decisions (participants were tasked with considering the likelihood of positive and negative health-related outcomes in specific scenarios). Only one of the four possible conditions resulted in optimal health decisions. That is, high numeracy abilities alone were not sufficient; alignment of both high confidence and high numeracy abilities were necessary to make optimal health decisions (Peters et al., 2019). Thus, best pedagogical practices should include an objective to ensure that the student's confidence and skills are aligned (Fives et al., 2014).

Numeracy Developmental Milestones

A large literature now supports the existence of two subsystems for representing quantities that are phylogenetically shared between human and non-human animals. However, each of these systems has limitations, as described below. Humans, even as infants, utilize the object tracking system (also known as subitizing) to precisely distinguish, without counting, small quantities up to four (Bull et al., 2006; D. C. Hyde & Spelke, 2011). In addition to subitization, infants can also approximate distinctions between larger quantities (e.g., six versus twelve) with the approximate number system (Halberda & Feigenson, 2008; Xu & Spelke, 2000). Infants are not alone in this ability to demonstrate numerical cognition without having acquired understanding of number words; a wide range of animal species have also shown sensitivity to object numerosity. For example, fish, birds, non-human primates, and equines also exhibit object tracking and approximate number abilities (Agrillo et al., 2014; Cantlon et al., 2016; Emmerton et al., 1997; Gabor & Gerken, 2014; Pepperberg, 1994).

These numerical capabilities observed across animal species and young infants are interpreted as support for the existence of systems that represent and process numerical quantities independently of

language (Halberda & Feigenson, 2008). However, another system is required to exactly represent quantities outside of the subitizable range (that is, quantities larger than 3 or 4). Most (but not all) human languages have a count list, that is, a sequence of words or signs that refer to the natural numbers (Butterworth et al., 2011; Corbett, 2000). In industrialized, numerate societies, children typically learn number words first as merely counting routines to be recited in sequence (similar to rehearsed games like "patty-cake".) Thus, young children (typically younger than age four) may appear to be able to count to relatively high numbers, such as twenty, but there is a dissociation between ability to recite number words in sequence, and comprehension of number word meaning (Fuson, 1991; Sarama & Clements, 2019).

While there is considerable variability in the timing, around the age of 2 years children learn that the linguistic symbol "one" refers to a single object (Carey, 2009). Upon achieving this developmental milestone, the child is then referred to as a "one-knower." In the classic "Give-a-Number" experimental paradigm, such children will correctly give one object when asked for one object, and will provide an incorrect number of objects when asked for two or any other larger quantity (Wynn, 1990). Children remain at this stage for multiple months before progressing to the two-knower level. The three-knower and four-knower levels subsequently follow, also with significant time elapsing between these stages. When the four-knower level is achieved children are generally able to successfully implement the cardinality principle. The cardinality principle refers to children's understanding that the last number word used in tagging a set of objects reflects a property of the set, and does not just apply to that object (e.g., Fuson, 1988). Once they learn the cardinality principle, they implicitly understand that when counting, (A) each word represents a specific quantity, (B) each object is labeled with a number word only once, and (C) each number word must be said in the correct order. As cardinal principle knowers, children are able to apply the cardinality principle to additional number words and rapidly expand their ability to represent larger quantities with precision (Carey, 2009).

Number Development and Language Experience

The nature of the relationship between numeracy and other domains of cognition remains under debate (Hohol et al., 2017). Numerical cognition research has often pointed to domain-specific systems as the basis for mathematical skills (Feigenson et al., 2004). As described above, the approximate number system is posited to be a lower-order, nonverbal process that scaffolds and accounts for higher-order numeracy outcomes. However, while the developmental sequences outlined above appear to be universal in order, the timing of these milestones varies across and within linguistic and cultural groups and effects persist across the developmental spectrum up to adulthood. Differences in early life experiences with pedagogical approaches (Pagliaro, 2010), specificity of the lexicon for number (Gordon, 2004; Pica et al., 2004; Spaepen et al., 2011), and parental expectations and use of number language (Elliott & Bachman, 2018) have all been identified as sources of these individual and group-level variations in numerical cognition. Thus, it is clear that mathematical cognition is not unitary and that multiple domains, language in particular, contribute to number cognition (Carey, 2009; Levine & Baillargeon, 2016). Here, the focus is on the connections between language experience and numerical cognition. The multiple ways language affects numerical cognition across a range of populations and their specific language experiences reveals a robust mechanism that supports numeracy skills in all children, regardless of their auditory status or the modality(ies) of their language(s).

Globally, comparisons between deaf, hard-of-hearing, and typically hearing students' numerical cognition have shown d/hh children lagging behind typically hearing peers (Gottardis et al., 2011; Traxler, 2000). For example, Kelly and Mousley (2001) find that d/hh and typically hearing college students' arithmetic skills are similar, however, typically hearing college students tended to perform better than d/hh college students when confronted with word problems. Kelly and Mousley utilized an experimental design that allowed them to dissociate several skills: reading, basic arithmetic, comprehension of numerical relationships, and motivation to solve problems. By

dissociating these factors, they interpret the performance differences between d/hh and typically hearing children as arising not from literacy differences, but instead from pedagogical differences and expectations. Pedagogical practices of teachers of d/hh students appear to focus on rote practice of arithmetic rather than problem solving, which requires sophisticated use of mathematical language (Easterbrooks & Stephenson, 2006; Kelly et al., 2003; Ottem, 1980). These delays have a cascading effect throughout development and throughout life: The proportion of DHH people employed in Science, Technology, Engineering, and Mathematics (STEM) disciplines is very small (0.13–0.19%) compared to that of the general population (11–15.3%) (National Center for Science and Engineering Statistics (US), 2011). While the importance of accessible linguistic input, whether signed or spoken, to language development and academic success has long been acknowledged, attention to the role of language in mathematics achievement has been underappreciated and under-researched. The number of studies exploring number concept development and mathematical achievement in d/hh children is small; further, such studies rarely report or consistently control children's language experiences. While Pagliaro and Kritzer (2013) have suggested that exposure to signing deaf parents or adults increases d/hh children's "incidental learning opportunities" at home and in school, the specific role played by language in the development of number concepts in d/hh children has not been examined systematically—see Gottardis, et al., (2011) for a discussion and Carrigan et al., (in prep) for a study design that dissociates hearing status and language experience. Recent work has increasingly focused on the importance of early language development in children's later academic success (Borgna et al., 2018; Dietz, 1995; Risley & Hart, 2006; Snow, 2002; Vukovic & Lesaux, 2013; Weisleder & Fernald, 2013). We advocate bringing this same approach to the study of number concept development and mathematics achievement in d/hh children, beginning as early in development as possible (Cohrssen & Page, 2016).

Number Development and Language-specific Impacts

While all observed children go through these knower levels in the same sequence, the timing of this developmental trajectory does slightly vary as a result of experiences with specific types of languages (Almoammer et al., 2013; Barner et al., 2009; Piantadosi et al., 2014; Sarnecka, 2014; Sarnecka et al., 2007). For example, grammatical number encodes numerosity and has been shown to affect the timing and trajectories of children's numerical development. Grammatical number in English is expressed by adding the plural morpheme "-s" to a noun (e.g., "cats"), making it a singular/plural language. Other classifications include non-singular/plural (e.g., Japanese and Mandarin Chinese) and singular/dual/plural, that is, making obligatory grammatical distinctions between sets of one, two, and more than two (Slovenian and Saudi Arabic). Children learning languages with only singular/plural marking remain one-knowers longer than children speaking singular/dual/plural languages, who move more quickly to (and stay longer at) the two-knower level. Children learning singular/dual/plural languages are argued to receive the benefits of more extensive grammatical number marking (Sarnecka, 2014).

Language that parents use with children involving counting or labeling sets of visible objects is related to (Levine et al., 2010) and indeed has been shown to be causal for (Gibson et al., 2020) children's later ability to connect the appropriate quantity with the number word, and to their understanding of the cardinality principle. Parental talk about sets involving 4 to 10 objects more strongly predicted children's later cardinal-number knowledge than did talk about smaller sets (Gunderson & Levine, 2011). Further, number language produced by preschool teachers is also related to the amount of growth in children's number knowledge over the school year (Klibanoff et al., 2006). Number words are not the only type of language that has a positive impact on the development of number concepts. Children in a Head Start program who received a dialogic book-reading intervention focused on mathematical language improved in their number knowledge more than a control group

who received regular instruction. Examples of mathematical language included words and phrases like “a lot,” “more,” “inside,” and “near” (Purpura et al., 2017). Notably, both signed and spoken languages employ linguistic devices (e.g., reduplication) to encode aspects of number, such as plurality and magnitude (Corbett, 2000; Kurz & Pagliaro, 2019).

Numeracy Development in Impoverished Language Environments

Up to this point, we have discussed why mathematics development is a key component of childhood learning; that is, it may be the strongest predictor of academic outcomes (Duncan et al., 2008). We have also discussed the various ways numerical cognition can be impacted by language experience. Now, we turn to discussion of the negative consequences on numerical cognition development when children grow up in impoverished language environments, particularly, d/hh children.

Studies assessing the impact of various proxies for impoverished language environments (e.g. socioeconomic status (SES), home mathematics environment, school-related metrics) consistently indicate the importance of mathematics talk in early life (even prior to formal schooling). Elliot and Bachman’s (2018) review argues that individual differences in early childhood mathematics achievement is primarily accounted for by the characteristics of parents, specifically, their mathematics talk, their mathematics practices, and views about mathematics concepts. Parental education, a component of SES measurement, has also been found to impact mathematical language use (Purpura & Reid, 2016). The relationship between parental education and mathematical language can be modeled as an indirect relationship, with parental views about schooling and the resultant home learning environment as mediating factors (Taylor et al., 2004). This view finds further support when assessing the mathematical talk used by parents (controlling for education attainment) with their 3 year olds or 4 year olds. When educational

attainment was included in the statistical model, the results point to a possible divergence between parents of different educational attainment. Parents with higher educational attainment appear to provide increasingly complex mathematics talk that sustains their children's development to a greater extent (Thompson et al., 2017).

While parental education and mathematical talk is consistently found to be correlated with differences in mathematics performance, the extent to which it impacts early childhood performance is dwarfed by the impacts of language deprivation often found in d/hh children. More than 90% of d/hh children are born to hearing parents (Mitchell & Karchmer, 2004). Many experience language deprivation in early life, missing exposure to language during a critical time of development. Language delays have a pervasive effect on children and can create difficulties in everyday functioning (Ching et al., 2010). Children learn a great deal of information via the incidental learning that occurs in their everyday life outside of school. Spontaneous conversations that occur within a family and routine information acquired throughout day-to-day activities are an important part of language acquisition and overall learning; d/hh children often miss this information. Indeed, deaf adults from hearing families report missing much of this kind of contextual learning in childhood (M. L. Hall et al., 2018). A good example of this is known as "dinner table syndrome." D/hh children often do not have access to the auditory cues that are used to inform turn-taking, attention shifts, and conversational interruptions that are typically used by hearing people (such as occurs at a family dinner). They also lose the ability to follow the conversation when trying to interject information; as a result, d/hh children miss the information that is shared and cannot benefit from these naturally occurring, daily incidental learning opportunities (Meek, 2020).

Language strongly predicts many aspects of cognitive development generally, as well as academic performance. Indeed, using the NICHD Study of Early Childcare and Youth Development data set, Pace et al. (2019) showed that "kindergarten language was the only

predictor of longitudinal gains both within and across [academic] domains." Deaf children with delayed language access exhibit execution function deficits (Botting et al., 2017; M. L. Hall et al., 2018). However, deaf children exposed to a sign language from birth had equivalent parent-reports of executive function compared to hearing children (Goodwin et al., submitted; M. L. Hall et al., 2019). Executive functioning has been shown to support hearing children's numerical development (Simanowski & Krajewski, 2019), and executive functioning in preschool-aged children predicts later mathematics achievement (Mulder et al., 2017; Usai et al., 2018). Therefore, delayed language has effects on other cognitive functions that support mathematics development. In the domain of numerical cognition itself, acquiring a counting sequence (such as "one", "two", "three", etc. in English) from a language model early in development appears to be crucial for developing certain types of number representations. Deaf adults in Nicaragua who have not attended school or become part of the Deaf community, and therefore did not learn the count sequence of Nicaraguan Sign Language (NSL), struggle to exactly represent quantities above five. These adults, called homesigners,[1] are unable to reliably generate or match sets containing five or more items or events (Spaepen et al., 2011). Even when a count sequence is available to be learned, early access to the number words or signs is necessary. Flaherty & Senghas (2011) found that NSL signers who began learning NSL after early childhood also struggled to exactly represent quantities greater than six. These observations are not limited to deaf people or those in emerging sign language communities. Typically hearing adults whose native spoken language does not have specific words that refer to exact quantities in this way also have difficulty matching and representing quantities larger than four (Gordon, 2004; Pica et al., 2004).

1. See Coppola (2020) for a brief profile of adult homesigners in Nicaragua and Nicaraguan Sign Language. It is important to note that, as in many other low-income countries around the world, only about 5% of deaf people in Nicaragua attend school, are exposed to language, and participate in the Deaf community.

In addition to the work by Kritzer and Pagliaro demonstrating the role of parent-child interactions, and of mathematics talk in particular, in supporting mathematical development in d/hh children, recent studies have focused systematically on the role of language experience more generally (e.g., Madalena et al., 2020). In the following section we review evidence from Coppola's Study of Language and Math project examining the impact of language modality (signed or spoken), as well as the impact of the timing of when language exposure begins. These studies include children in the U.S. ages 3 to 9 years who are acquiring spoken English and/or ASL either from birth (typically hearing children and d/hh children who have at least one parent who is Deaf and signs ASL). To understand how the age of first exposure to language influences the development of numerical cognition, these early-exposed children are compared with d/hh children who begin acquiring one or both of those languages at some point later in development (i.e., upon receiving hearing technology such as a cochlear implant or hearing aid, or upon entry into a signing educational program). Rather than focusing on how language is used in the home or school contexts, these studies focus on the symbolic role of language itself (e.g., Carey 2009), and on when a child begins acquiring language (e.g., Mayberry, 2010; Newport, 1990), in developing foundational number concepts. These studies provide additional evidence supporting the critical role of early exposure to spoken or signed language, in particular the count sequence, in the development of quantity representations. The studies address the impact of language experience on symbolic number representations that clearly depend on language, such as the meaning of the sign for "seven," as well as on non-symbolic number representations, which have been held to not depend on language, such as the ability to precisely track small quantities of objects and the ability to approximately discriminate large sets of objects.

As described earlier, mastering the cardinal principle marks a milestone in children's number development. The Give-a-Number task is widely used to measure children's understanding of, rather than

mere recitation of, number words, such as "one, two, three." Recall that children who can accurately create sets of four items upon request (as well as all of the quantities below 4) are generally considered to know the cardinal principle and be able to create accurate sets for all of the numbers in their count list. Studying 55 children with typical hearing and 121 d/hh children who were learning spoken English and/or ASL, researchers in the Study of Language and Math found that the timing of language exposure and age each independently predicted whether a child had mastered the cardinal principle, but language modality and socioeconomic status did not. Older children, and those who were exposed to either language from birth, were more likely to demonstrate understanding of the cardinality principle than children who were exposed to language later in development (Carrigan et al., in prep.; Contreras et al., 2019).

A subsequent milestone in the development of number cognition is "mapping," the ability to rapidly and automatically translate among number representations (e.g., Arabic numerals, signed and/or spoken number words, and dot arrays), which predicts children's later academic success (Brankaer et al., 2014; Göbel et al., 2014; Mundy & Gilmore, 2009). In a study of 142 d/hh children and 48 typically hearing children ages 5 to 9 years, (Walker et al., 2021) asked whether the modality and timing of children's language exposure influenced their mapping skills. In this task, children saw a target quantity expressed by dots, a signed or spoken number word, or an Arabic numeral, and then had to point to the matching quantity or symbol from an array of four options. Contrary to arguments that deafness itself delays mathematical abilities, the early-exposed children, that is, the d/hh children who were exposed to ASL from birth, showed similar performance as typically hearing children exposed to spoken English from birth. Further, the early-exposed children performed better than d/hh children with delayed language exposure to either spoken or signed language. These results suggest that early access to language is critical for mapping, an essential prerequisite of calculation fluency (Walker et al., 2021) and mathematical development.

Recent findings also suggest that, contrary to the widely-held view, language experience may influence development of the approximate number system (Santos et al., in prep, 2019). When asked to point to "which of two sides of a computer screen contained 'more' dots," d/hh children between the ages of 3 and 6 years who were acquiring only spoken English via cochlear implants (n=14) (i.e., no experience with a signed language) showed lower accuracy than did typically hearing children (n=45). However, these d/hh children performed comparably to the typically hearing children on a version of the same task that used the same dot displays, but which did not use any linguistic instructions. Further, when the ages of the d/hh children were adjusted to account for their later access to spoken English, (i.e. their "language age"), their performance was similar to that of younger typically hearing children who had been exposed to language for the same amount of time (Santos et al., 2019).

Indeed, Santos et al. (in prep) is the first systematic exploration of how language experience—namely the timing of language input (beginning at birth vs. beginning later in development) and language modality (spoken English vs. ASL)—may influence the development of approximate number system acuity in d/hh children. In a study of 200 children, they found that children who were exposed to spoken or signed language later in development (n=90) showed poorer approximate number system acuity than did children who were d/hh and typically hearing who began learning their first language at birth (n=110). However, their task instructions were linguistic, leaving open the possibility that language experience affected children's understanding of the task, and not their actual approximate number system acuity. A subsequent analysis indicated that most of the children showed patterns of performance indicating that they did understand the task (i.e., they performed better on trials with larger differences between the two set sizes (e.g., 9 vs. 3 items, a 3:1 ratio) than they did on trials in which the two sets were closer in quantity (e.g., 13 vs. 10 items, or a 1.3:1 ratio).

The other subsystem for representing quantity that has not been thought to depend on language is the ability to track small quantities of objects. Quam et al. (under review) adapted the "Mr. Elephant" task (Shusterman et al., 2017) in which an experimenter placed "peanuts" (actually small balls) into a large wooden elephant toy, and either some or all of the balls exited the toy. Quam et al. analyzed two trials: one in which two balls went in and two exited, and one in which three balls went in but only two balls exited. In each trial, the child had to say whether all of the balls had come out, or whether any were still stuck inside. The timing of language exposure and socioeconomic status significantly predicted Mr. Elephant performance, while language modality and age did not. Later-exposed children (n=69) were less likely to succeed on the task than Early-exposed children (n=84). An exploratory follow-up analysis included two measures of language: Highest Count, which records how much of the count list children can recite and Give-a-Number (described above), which assesses children's understanding of number word meanings. In this model, the timing of language exposure and Give-a-Number performance significantly predicted children's performance on Mr. Elephant, but socioeconomic status and Highest Count did not. That is, children's actual understanding of the quantities referred to by the words in the count list, but not rote recitation of the count list, affected their ability to accurately track small quantities of objects. Taken together, such findings are suggestive that language is important for the development of non-symbolic representations of quantity, which have historically not been considered to rely on language. It is possible that a certain amount or type of language input or experience is required to support the development of both approximate number system and small-object-tracking abilities; the reason that the influence of language has not been detected in previous work might be that children who are typically hearing are practically guaranteed to exceed that threshold, while some d/hh children may not.

All together, these findings, combined with the suggestions by Gottardis et al. (2011) and others, indicate that studies reporting

poorer mathematical abilities in d/hh children suffer from a serious confound, namely that they fail to account for the documented variations in d/hh children's language experiences and the implications for the development of their number representations. Notably, these variations in d/hh children's language experience appear to exert greater influence on mathematics outcomes than the variations observed in children from high and low SES environments. As seen above, delayed language exposure can result in poorer performance on tasks measuring acuity of the approximate number system and the object tracking system, whereas differences in mathematics performance attributable to SES seem restricted to verbal tasks (e.g. story problems, Jordan & Levine, 2009). SES-attributed differences in mathematics performance on non-verbal tasks (e.g. non-symbolic magnitude representation) are not typically found to differ between lower and higher SES groups (Jordan & Levine, 2009). Here, we propose that the most effective investigation of numeracy development is optimally made with consideration of how some kinds of number representations depend on language. By examining the effects of language deprivation (W. C. Hall, 2017) on numerical development, we can better understand the development of numerical cognition and children's outcomes across a broad spectrum of populations (e.g., bilinguals with later formal written language instruction in mathematics, children with significantly impoverished signed and/or spoken language input, like d/hh children). With this improved understanding, we can implement targeted interventions and make more effective curriculum policy decisions.

Language and Pedagogical Approaches

While national and state-level standards establish targets for P-12 mathematics learning, day-to-day implementation of pedagogical strategies are made by teachers and there are a multitude of pedagogical philosophies and approaches in use (Easterbrooks & Stephenson, 2006). These different practices have received considerable attention, though they remain woefully understudied,

with nearly all identified approaches classified as requiring additional empirical research to properly test their claims (Beal-Alvarez & Cannon, 2014; Easterbrooks & Stephenson, 2006). Easterbrooks & Stephenson (2006) identified 10 different practices relevant for science and mathematics instruction, which can be categorized into four themes, (a) language, (b) instructor expertise, (c) critical thinking, and (d) technology and supporting materials. The Easterbrooks & Stephenson (2006) article is complemented by a second publication presenting a survey of master instructors and their evaluations of the importance of the same ten practices (Easterbrooks et al., 2006). Nearly all master instructors surveyed agree that high skills in communicating science and mathematics are of paramount importance. In addition to the perception of the importance of language by master instructors, discussion of pedagogical practices relating to language have received tremendous attention. Careful assessment of efficacy, however, is sorely lacking.

Nunes and Moreno (2002) and Nunes (2004), Zarfaty et al. (2004) recommend a mathematics curriculum that emphasizes visual-spatial representation for deaf preschoolers. In their study with sets of visual brick construction, 3- and 4-year-old deaf children outperformed their hearing peers in the spatial tasks and performed on par in the temporal task. Building on deaf children's strengths, one of which is visual skills, and supporting their numeracy trajectory progression, the teacher is encouraged to include components of visual-spatial pedagogy, such as object teaching and the use of multiple visual representations, in the preschool classroom (Nunes et al., 2006). Given that development of foundational numerical concepts and learning of higher-order arithmetic processes are influenced by language, findings that better mathematical performance was observed in children with robust access to language is unsurprising. D/hh students who were unable to understand spoken presentation of instructional content were unable to solve the prompted mathematics questions (Enderle et al., 2020; Serrano Pau, 1995). Deaf children need a foundation in a fully accessible language for literacy

(W. C. Hall, 2017) and numeracy development (Pagliaro, 2015). Teachers' language skills are crucial to providing an accessible language to deaf children. The role of language fluency of the instructor is further elevated considering that d/hh children have fewer opportunities for incidental learning because relatively few of their parents are fluent in a signed language (Nunes & Moreno, 1998). The teacher must then be capable of producing a fluent utterance that not only conveys meaningful mathematics concepts (Schindler & Davison, 1985), but also offers opportunities for deaf children to develop informal mathematics knowledge, such as numbers, time, sequences, and categorization, and to develop language skills in early grades (Pagliaro & Kritzer, 2013).

Teacher knowledge of early mathematical concept learning trajectories may affect the teacher's ability to teach to deaf young children (e.g., Pagliaro & Kritzer, 2013). Understanding mathematical learning milestones as children progress from early childhood to elementary grades benefits lesson preparation, implementation and reflection. Lacking understanding of these milestones can lead to lower teacher expectations and instructional rigor (Pagliaro, 2006; Wilson et al., 2014). Teacher training programs that prepare teacher candidates for early childhood programs and primary grades should incorporate learning trajectories of early mathematical concepts, and the role of language in these developmental trajectories, as pedagogical tools (Wilson et al., 2014).

Number and Mathematics Interventions

Researchers and educators have developed multiple practices for preschool and elementary school children to improve their number knowledge and mathematical performance (without regard to deafness and its contextual factors) (c.f. Frye et al., 2013). Further studies assessing the efficacy of numeracy interventions, especially with very young d/hh children who have not yet begun formal schooling, are very much in need. Following up on work showing that d/hh children begin kindergarten without the requisite numeracy

foundation (e.g., Kritzer, 2009) carried out an intervention, building the Math Readiness: Parents as Partners (MRPP) project, designed to increase parental behaviors that are known to support the development of foundational mathematics concepts (Kritzer & Pagliaro, 2012; Pagliaro & Kritzer, 2013). Their efforts to train parents to mediate early mathematics concepts with their d/hh children were successful in increasing parents' use of mathematics and related vocabulary and in other measures of parent-child interactions. However, the causal impact of specific parental behaviors on the children's mathematics performance was not evaluated.

Parent mathematics talk can also have a positive and causal impact on children's number knowledge (Gibson et al., 2020). Gunderson and Levine (2011) found that number knowledge in children is predicted by their early experiences with number language, specifically their understanding of the cardinal meanings of number words (e.g. knowing that the word "three" refers to a set of three items). Susperreguy and Davis-Kean (2016) found a positive correlation between mathematics talk by parents and their child's mathematics abilities, even a year later. Zippert et al. (2020) found that when providing math-related tablet computer games, parent-child mathematics talk can be enhanced by providing brief parental support and guidance. These correlations are suggested to arise from parental mediated exposure to behaviors that facilitate learning, which provides children with effective tools to incorporate new information into existing knowledge (Pagliaro & Kritzer, 2010) The d/hh children in Pagliaro & Kritzer's study who were exposed to and applied greater numbers of learning strategies in with social interactions also had higher mathematical abilities. For example, when the parent and/or child (a) initiates communication with a statement, then (b) questions "why," and then (c) offers an explanation or response, they are applying three distinct behaviors that can prompt learning that will support the integration of knowledge. Using these mediated learning strategies along with mathematical language is also correlated with mathematical skills (Kritzer, 2008).

Kritzer found higher mathematical ability was correlated with frequency of exposure to number in a range of contexts (i.e., counting, quantities, time and sequence, and categories). Parent mathematics talks can be duplicated in the early grade classroom where d/hh children are exposed to accessible languages that might be lacking in home environments. The teacher as a meditator provides structured and unstructured mathematics talks to promote numeracy development in early d/hh learners.

Following recent findings from studies summarized above that demonstrate associations between d/hh children's language experiences and the development of number concepts (Carrigan et al., in prep.; Walker et al., 2021), one planned project in the Study of Language and Math (Coppola, 2016) was designed to evaluate the causal impact of parent language, specifically use of the counting sequence, on d/hh children's number development. The goal of the study, which was unfortunately interrupted by the Covid-19 pandemic, was to examine the impact of dramatically increasing the amount of number words and counting behaviors experienced by d/hh children between the ages of 2.5 and 5.5 years who were acquiring spoken English via hearing technology (Coppola, 2016). In this design, parents read specially designed number books with their children, and encourage the children to count sets of objects ranging from 1 to 10. These children will be compared to similar children who have been randomly assigned to read books that focus on the associations between letter names (e.g., "bee") and the shapes of letters (e.g., B). Outcome measures include the difference between pre- and post-training performance on the Give-a-Number task (creating sets), as well as the Which-is-X task, in which children see two sets of the same type of object (e.g., four birds vs. five birds) and are asked to "point to four." If exposure to number words and counting itself drives the development of number knowledge, including cardinality, Coppola and colleagues hypothesize the children in the number-book reading condition to show greater growth relative to children in the letter-book group (controlling for other factors that

are known to influence number knowledge growth, such as age, general vocabulary, executive functioning, and socioeconomic status). One way to gather the volume of data required to support the development of evidence-based practices is to develop techniques to widen the scope of participation in such studies to a national level, and also to include d/hh children who are acquiring signed and/or spoken language. One example of this type of approach would be developing a website to encourage parents across the U.S. to engage their preschool-aged d/hh children in more everyday interactions that involve number, and to increase their use of mathematical-related language with them (an approach already used on a smaller scale by Pagliaro and Kritzer). These interventions leverage what is already known about the power of early number knowledge to influence and predict later academic outcomes.

Summary

Here, we have provided a succinct overview of the major elements of numeracy development, from (purportedly) non-linguistic representations of small precise and large approximate quantities to language-dependent and precise representations of large quantities. This process is not straightforward and rapid: in fact, it typically requires children 2-years old to be able to decipher the rules underlying the cardinality principle and then apply the cardinality principle to numbers greater than four. Strong numeracy skills consistently predict positive academic outcomes, pointing to the need for prioritizing numeracy instruction time to help ensure all children have a strong foundation in numeracy skills.

As shown from our review of the literature, the past 20 years highlight the impact of language experience and children's linguistic fluency on their numeracy foundation in preschool and their later academic outcomes. Some authors have articulated a rights-based argument for putting into place early childhood education practices that provide all children with a solid foundation in numeracy (see Cohrssen & Page, 2016). Based on this literature, we describe policy

and curriculum practices that optimally support the development of numerical cognition. These practices can be further informed by ongoing research that will elucidate the underlying mechanisms of numerical development in very young and preschool-aged children, as well as specific pedagogical practices that can foster such learning in older school-aged children. As Draper stated 144 years ago, and which is still relevant now, every child today has a natural and innate curiosity about the world and they need to be provided with natural, structured and unstructured, consistent and sporadic activities at home, in school and elsewhere, to build mathematical knowledge. Natural language learning interactions and practices reinforce their early mathematical concepts, including number sense.

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