NCTM ILLUMINATIONS

illuminations.nctm.org

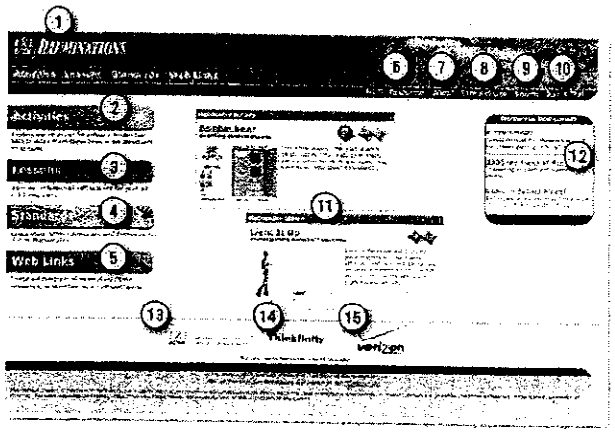
Illuminations provides standards-based resources and materials that illuminate the vision of NCTM for school mathematics and improve the teaching and learning of mathematics for all students. Lessons and activities on the Illuminations site have been developed in alignment with NCTM's *Principles and Standards for School Mathematics*.

Using Illuminations

From the Thinkfinity homepage, click on the Partner site logo, or type **illuminations.nctm.org** in the address line of your Web browser, and click Enter or press Return.

The Illuminations homepage appears.

The following numbered list corresponds to the screenshot on the right showing the features of the homepage.



(1) Visit **Illuminations** to find mathematics resources designed to improve teaching and learning.

(2) Watch mathematics come alive in the **Activities** section, which contains interactive online activities for both teachers and students.

(3) Search the **Lessons** section to find single-day lessons or multi-day units in mathematics for grades pre-K–12.

(4) Learn about NCTM's *Principles and Standards for School Mathematics* in the **Standards** section.

(5) Visit the **Web Links** section to find the best online destinations for teaching and learning mathematics.

(6) Browse NCTM's catalog of math-related **NCTM Resources**.

(7) **About** provides background information on Illuminations and NCTM.

(8) Teachers can use Illuminations materials freely in their classrooms, but all other uses are governed by NCTM's **Terms of Use**.

(9) Search Illuminations and all of the Thinkfinity Partner sites.

(10) Join NCTM to become part of a dynamic group of over 100,000 math education professionals.

(11) Updated regularly, the **Highlighted Activity** and **Highlighted Lesson** showcase some of Illuminations most popular resources.

(12) Learn more about NCTMs **Professional Development** opportunities.

(13) Visit **nctm.org**, the official Web site of the National **Council of Teachers of Mathematics**.

(14) Access the **Thinkfinity** Web Site.

(15) Access the **Verizon Foundation** Web site.

NCTM Mission Statement

The **National Council of Teachers of Mathematics** is a public voice of mathematics education, providing vision, leadership and professional development to support teachers in ensuring mathematics learning of the highest quality for all students.

Illuminations – Overview

Content Area(s) Mathematics
Sponsor: National Council of Teachers of Mathematics

Major Resources

- **Lessons**
- **Web Links** –divided into 5 Content Areas: Number & Operations, Algebra, Geometry, Measurement, Data Analysis & Probability
- **Standards** – 5 Content standards, grouped by grades with electronic examples
- **Activities** – more than 60 online activities

Search For	By	From
Lessons	• Grade Level • Standards • Keyword • Online Activities	Lesson page
Web Links	Not searchable, organized within 5 content standards	Web Links page
Activities	• Grade Level • Keyword	Activities Page

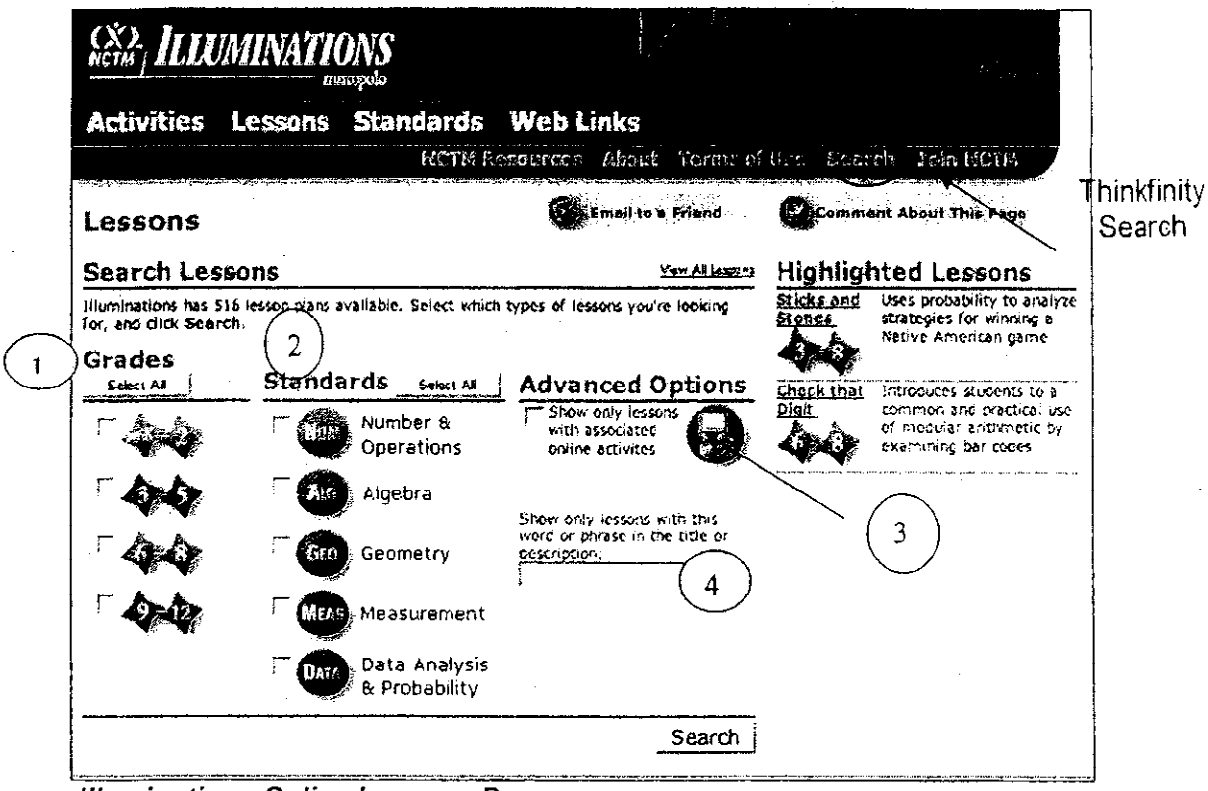
Site Strengths

- There are many online activities. If they are in a lesson you can link directly to it from the activity page.
- Standards have Electronic Examples and Video*
- Search of lessons by online activities
- Each web link that is listed is reviewed by a panel and categorized into reference, tool, lesson or activity tool.

Technology Integration
The Lesson, Activities and Web Links are all inter connected.

*Some videos need Apple Quicktime

Illuminations Lessons



Illuminations Online Lessons Page

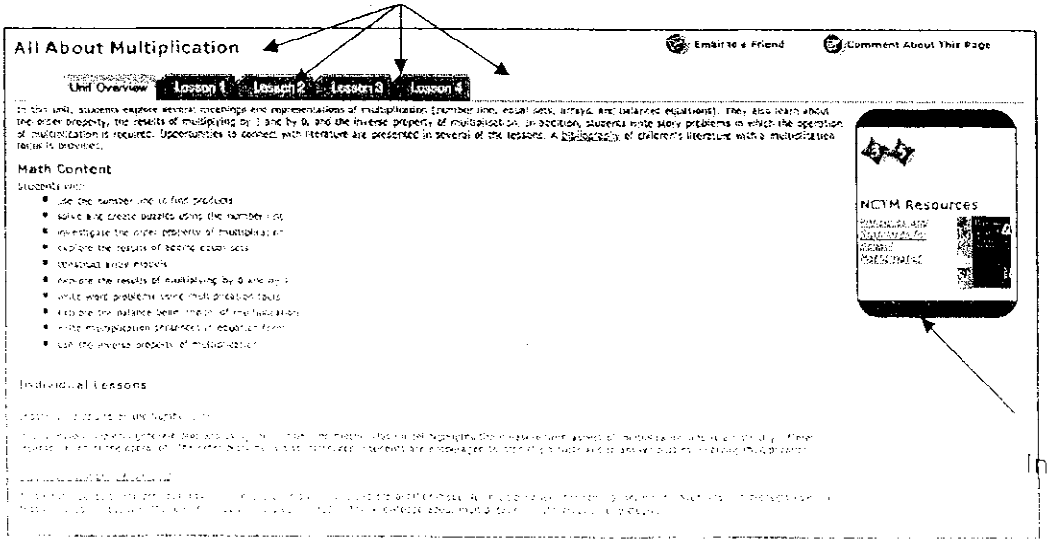
Searching for Lessons:

You can search for lessons within the Illuminations by:

1. Grades
2. Standards
3. Online Activities
4. Keyword

Overview of Lessons Pages

Some of the Lessons have multiple lessons for the Topic you choose. You can access the Unit Overview or the individual from the Tabs at the top or the links below.



Lesson Page:

Shorts and Shirts

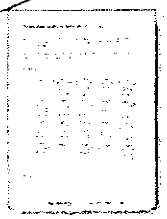
Students are encouraged to discover all of the combinations for the given situation. Students apply problem-solving skills (including elimination and collection of organized data) to draw their conclusions. The use of higher-level thinking skills (synthesis, analysis, and evaluations) is the overall goal.

Learning Objectives

Materials

Instructional Plan

Distribute the Shorts and Shirts activity sheet to each student. Students will be coloring outfits to determine the total number of combinations possible.



Shorts and Shirts Activity Sheet

Give each student eight crayons: red, green, yellow, blue, orange, brown, black, and purple.

Review the problem by reading it with the students:

Each shirt must be a solid color, either yellow, orange, blue or red. Each pair of shorts must be a solid color, either brown, black, green, or purple. How many different outfits can be made? No two outfits should be the same.

1 period

NCTM Resources

Navigating through Data Analysis and Probability in Grades 3-5

Activities

- Bobbie's Bear

Worksheet Link (PDF)

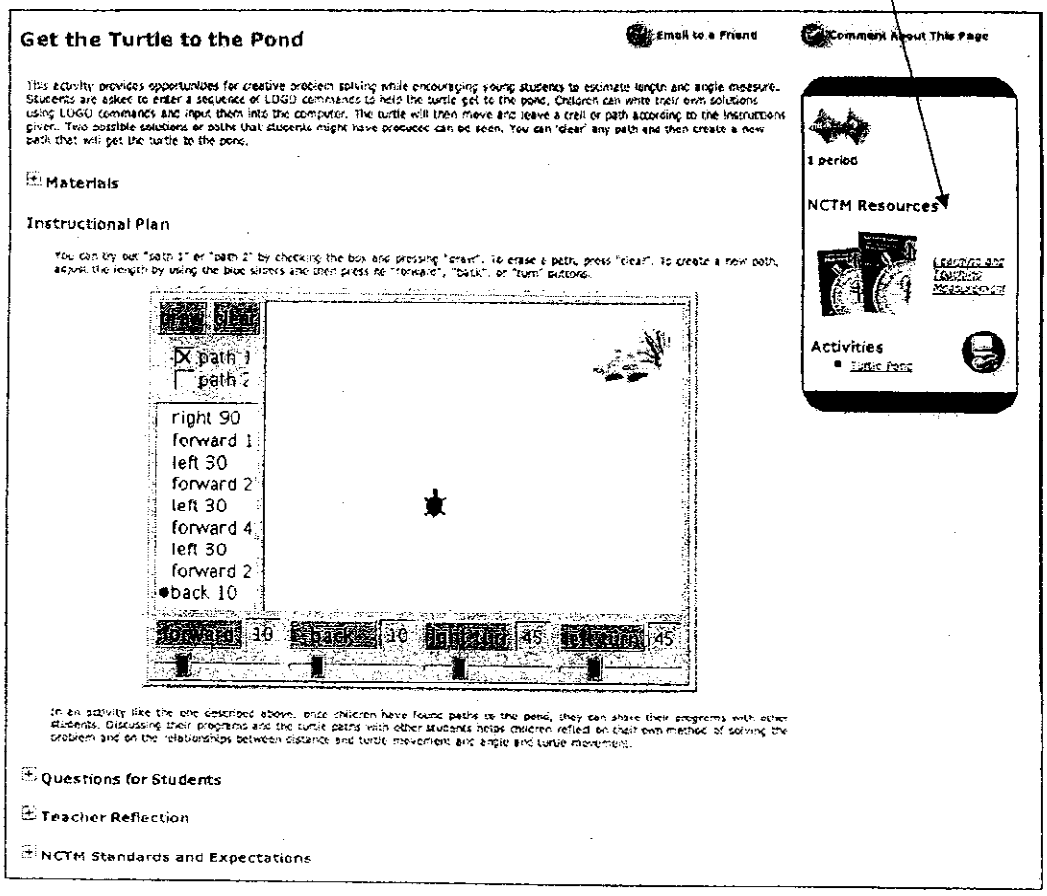
Related Informatio

Parts of Lessons	
Learning Objectives	Always included but not always labeled
Materials	Lists the materials needed for lesson
Instructional Plan	• Plan for activity • Use the online activity within lesson or click in Related Information box to just use the activity • May include worksheets accessible by Adobe Reader
Questions for Students	Not always included Sometimes with answers
Assessment Options	Sometimes with a link to a sheet or website
Extensions	More ideas through web sites on the topic
Teacher Reflection	Questions for teachers to reflect on after the lesson is completed
NCTM Standards & Expectations	Always included
References	If outside references are used, it is cited here
Student Pages	There are links to student worksheets or sites but no specific student, teacher pages
Related Information Grade Level Time Frame NCTM Resources Activities Lessons Web Sites Other	In box on right side of screen, Grade level and NCTM resources always display Time given in periods Books related to topic for purchase through NCTM Links you directly to Activity used in Lesson Displays links to other lessons with similar topic Links for web sites on the topic • Buttons to email lesson • Button to comment on lesson
Author	Sometimes included

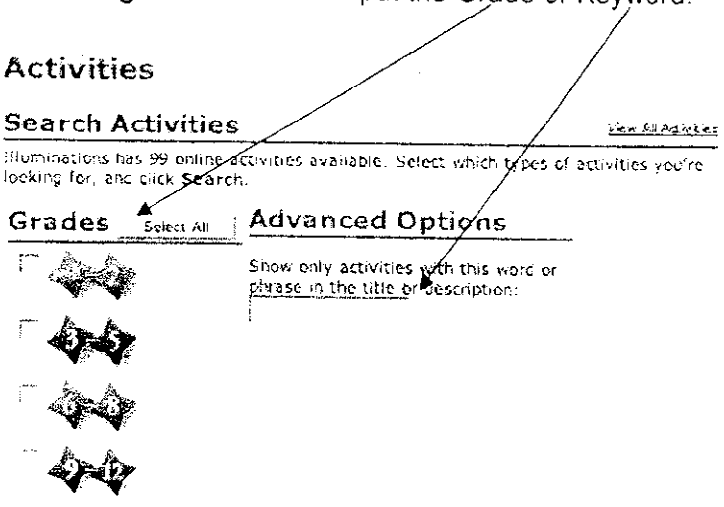
Sample Lessons		
Title / Grade	**Search – Lesson Page**	**Comments**
Get the Turtle to the Pond, PreK-2	PreK-2, Geometry content	Could use just the Activity on right
The Product Game 3-8	Number Content, Online Activities	• Displays 4 Lessons • In Lesson 4 there is a Product Game that 2 kids can play together
Transformation & Frieze Patterns 9-12	Online Activities, keyword – symmetry	• Includes worksheets, online activities and websites • Nice visual on the manipulatives to understanding symmetry
Combinations 3-5	Data Analysis content, Online Activities	• Includes 2 lessons with worksheets and activities • Lesson 2 has Bobbie Bear applet for students to create clothes combinations

Activities

Activities are the online student materials in Illuminations. These links do not give a full lesson plan just an activity on a topic. They are broken into Instructions and Explorations. There is a related information box with the NCTM Resources and Lesson links.



Searching for Activities: Input the Grade or Keyword.



Sample Activities		
Name	**How do I Find?**	**Comments**
Turtle Pond	PreK - 2	Teaches creative problem solving and measurement through LOGO commands.
Fraction Model 1	Keyword - fraction	Allows students to see different representations of fractions by visually seeing the change in numerator or denominator.
Mean and Median	Keyword – mean	Visually display a box-and-whisker plot with data a student creates.
Product Game	Keyword – factors	This Activity is in Lesson Playing the Product Game
Factor Game	Keyword – factor game	• Play against the computer or a friend • Helps students know the factors of numbers
Adjustable Spinner	Grades 3-5	Shows probability on multiple spins

Web Links

Web Links are links to specific sites that fall into one of the content areas. You have access to thousands of web sites that have been reviewed and approved by a panel. When you select the name of the Web Link you go to the review page. The page lists the grade, standard, type of resource (online, lesson), a review and a description of the owner of the page. You can skip the review and go directly to the web link by clicking on **Direct to Web Resource.**

Searching for Web Links: You cannot search this site they are organized in five categories of content areas: Number & Operations, Algebra, Geometry, Measurement, Data Analysis & Probability.

Sample Web Links		
Name	**How do I Find?**	**Type/Comments**
Math Forum - Arithmetic	Number and Operations	**Reference** Collection of classroom materials, software, internet projects and public forums for discussion related to Arithmetic
Tour of Fractions	Number and Operations	**Reference** Collection of lesson plans, and software covering all topics related to fractions.
Icon: Figure This!	All contents	• When you see the Figure This icon you can get to challenge problems • For all the content areas
Quick Math	Algebra	**Tool** Solves common math problems
Pie Chart Virtual Manipulative Icon:	Data Analysis & Probablility	**Tool** Contructs pie charts showing values and % relationships
Parts of a Whole http://nlvm.usu.edu/ Using this web link to get to this site OR just type in the URL	Geometry	Brings you to a Web Link review that is not active but if you click on the site for Manipulatives: http://matti.usu.edu/nlvm/nav/index.html It is a great site for all the content areas – click on the grade and content you would like to explore. Example • grade 6-8, geometry on the grid • Transformations - Reflections

Teaching Scenario: Elementary

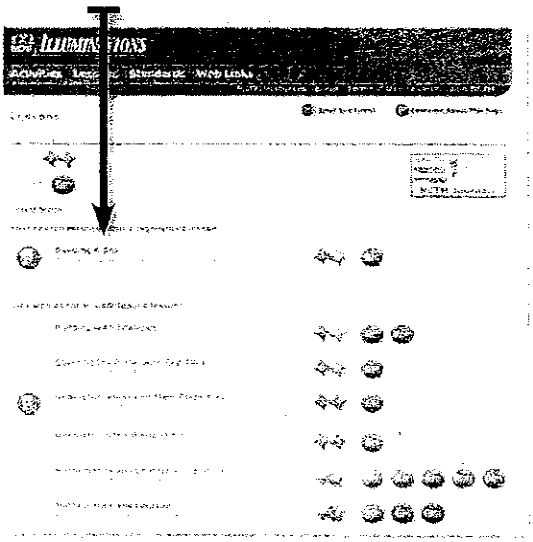
illuminations.nctm.org

Setting the Stage

Ms. Martinez is about to begin a geometry unit with her class of fourth graders in San Diego. In the past, she has noticed that her students had difficulty with spatial visualization, the ability to predict what things will look like in three dimensions. Her school uses textbooks and workbooks that contain good illustrations, but based on her experience and a review of educational literature, she believes that students will develop stronger spatial visualization skills if they use hands-on, interactive activities. In addition, *Principles and Standards for School Mathematics* suggests that spatial visualization and reasoning are core skills that all students must develop, and the California State Content Standards for Grade 4 note that students should be able to "visualize, describe, and make models of geometric solids (e.g., prisms, pyramids) in terms of the number and shape of faces, edges, and vertices; interpret two-dimensional representations of three-dimensional objects; and draw patterns (of faces) for a solid that, when cut and folded, will make a model of the solid." The full text of the California Standards are available at *http://www.cde.ca.gov.*

Ms. Martinez's classroom has 10 computers with Internet access. There are 30 students in her class. This allows her to divide the students into groups of three for computer work.

(1) From the Illuminations home page, Ms. Martinez chooses **Lessons** from the main toolbar and then she searches for **Geometry lessons in Grades 3-5**. Although she gets many results, she decides to investigate the highlighted **Building a Box** lesson, because the description indicates that it will develop students' spatial visualization skills.



(2) When Ms. Martinez looks over the lesson, she especially likes that students are asked to construct boxes. She believes that this hands-on approach will go a long way in helping her students see what happens when a 2-D figure is transformed to a 3-D object.

(3) She also likes the story on the **Building A Box activity sheet** that accompanies the lesson (see the story's text below):

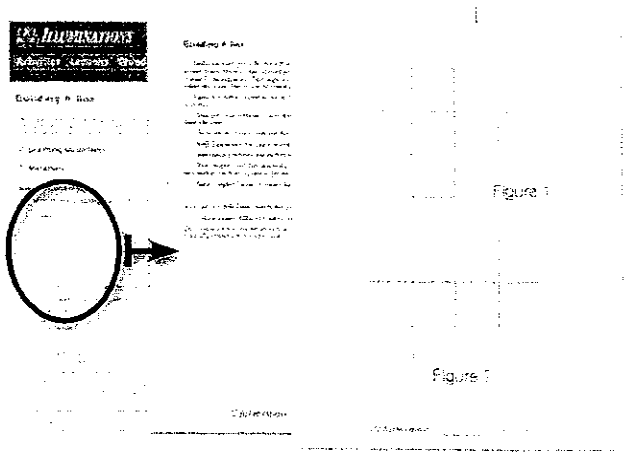


Figure 1

Figure 2

Emma got a new job at the Acme Box Factory. Her job is to construct cubes that will be used as jewelry boxes. Her boss, Ron, showed her the company's current blueprint for making these boxes (Figure 1). He explained, "This shape is called a net. A *net* is a flat figure that can be cut out and folded into a box. This net can be folded into a cube that measures 3 centimeters on each side."

Emma was then instructed to cut out Figure 1 and fold it into a cubical box. *(You may also want to do this.)*

"Your job," Ron continued, "is to draw as many of those nets as you can, cut them out, and fold them into cubes."

"Do all my nets have to look like this one?" asked Emma.

Teaching Scenario: Elementary *continued*

"Well, I guess they don't have to look like that... but how else could they look?" inquired Ron.

Emma quickly sketched out another net (Figure 2) and exclaimed, "Wouldn't this also work?"

"Yeah, maybe," said Ron skeptically. "It doesn't matter to me how you do it. You can make the nets anyway you want, as long as you end up with cubes measuring 3 centimeters on each edge."

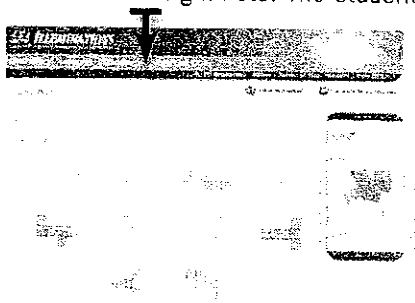
"Great!" replied Emma. "I wonder how many ways there are to make such a net?"

Your task is to help Emma answer this question:

How many different nets can you draw that can be folded into a cube?

Ms. Martinez decides to teach this lesson, but instead of giving students the activity sheet, she copies it onto a transparency and displays the story on the overhead projector. She draws a grid of 3 cm × 3 cm squares on a sheet of blank paper, copies it, and distributes several sheets to each student. Students can then use these sheets to test various nets.

(4) As it turns out, there are 11 unique nets that will form a cube when folded. Students in Ms. Martinez's class have varying levels of success in finding them. Some students were able to find only five or six, and one student found nine different nets, but no one found all 11. Ms. Martinez divides students into groups of three and has them compare their results. One student who found only five nets is grouped with another student who found eight nets. The student with only five found two nets not discovered by the other student, so combined they had found 10! Still, none of the groups were able to find all 11.

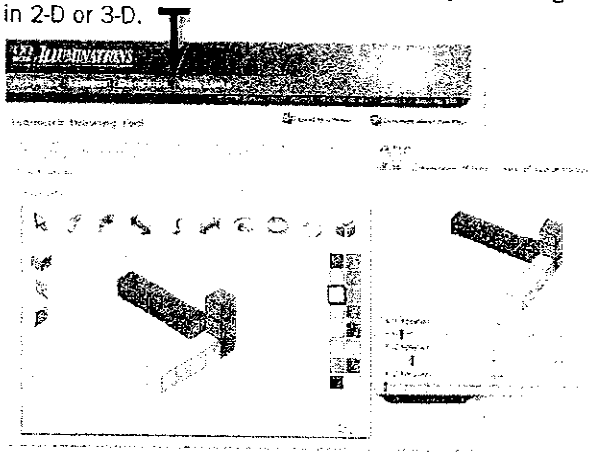


"I believe that math teachers struggle the most when finding ways to integrate technology [in the classroom] with their students. [Illuminations] has an answer for all math topics and levels and integrates technology in many of the lessons. The Activities section is outstanding!"

— Julie Turner
District Instructional Technologist: Boerne, Texas

Ms. Martinez allows students to self-assess their work with the **Cube Nets** activity, which she finds through a link at the bottom of the **Building A Box** lesson plan. Again working in groups of three, students use this tool to discover all 11 nets. In particular, they are to identify those nets that they hadn't discovered during the lesson.

(5) The students really enjoy using the **Cube Nets** activity, so Ms. Martinez looks into using other NCTM resources by browsing through the interactive applets available from the **Activities** section of Illuminations. This led her to the **Isometric Drawing Tool**, which allows students to create 3-D figures using cubes. Students can shift, rotate, color and decompose figures and they can view any of the figures in 2-D or 3-D.



The six lessons from the **Using Cubes and Isometric Drawings** unit provided Ms. Martinez with a lot of good ideas. In particular, she liked the ideas in the lesson, **Building Using the Front-Right-Top View** located within the **Lessons** section. Using these views to create a 3-D shape—or, conversely, drawing the front, right, and top views of a 3-D object—falls exactly in line with her state requirement that students be able to interpret two-dimensional representations of three-dimensional figures.

Teaching Scenario: Secondary

illuminations.nctm.org

Setting the Stage

Ms. Naylor is a middle grades teacher in Pennsylvania. On the first day of school, she asks students to determine the value of the following expression:

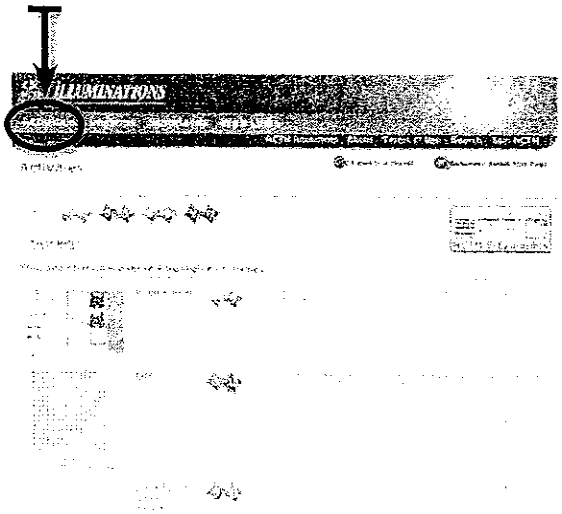
$$5 + 10 \times 2$$

Students excitedly call out the answer, "30!" But then Ms. Naylor asks them to enter the expression into their scientific calculators, and another answer is returned: 25. Confused by the results, students are interested to learn why the answers are different. This leads to a discussion of the order of operations.

Ms. Naylor knows that understanding the order of operations is important for students, because it will lead to their later success in algebra. Moreover, she knows that Pennsylvania students will be tested on their ability to "simplify numerical expressions involving exponents, scientific notation and using order of operations." From her many years in the classroom, however, she knows that students are often reluctant to write out all the steps when determining the value of a complex expression. She turns to Illuminations to find a way to convince students that writing out each step is necessary. The full text of the Pennsylvania Standards is available at *http://www.pde.state.pa.us*.

Ms. Naylor has a computer connected to a TV in her classroom, and she is able to project what she sees on the computer screen to the entire class. With advanced planning, she can also schedule time for her students in one of the school's two computer labs.

(1) Ms. Naylor first learned about applets when she was introduced to the E-Examples on NCTM's Standards Web site, *http://standards.nctm.org*, a few years ago. Since then, she has become a regular visitor to Illuminations, because it contains over 60 applets in its **Activities** section, each of which can be used to demonstrate various mathematical concepts.

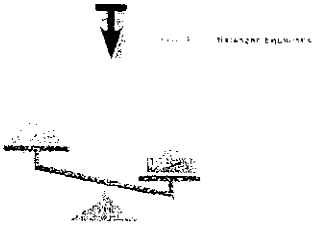


(2) In looking over the list of Activities, she found the Pan **Balance – Numbers** activity, which allows an expression to be entered into each side of a pan balance. If the expressions are equivalent, the pans will balance. Using this tool, she entered $5 + 10 \times 2$ into the red pan and the number 25 into the right pan, and the pans balanced. She then replaced the 25 with the expression $5 + 20$, and again the pans balanced. In the Balanced Equations area, she saw the following:

$$5 + 10 \times 2 = 25$$
$$5 + 20 = 25$$

She then realized that, by using this applet with her students, she could teach them the order of operations and encourage them to write out each step when simplifying complex expressions.

(3) In class, she projected the Pan Balance – Numbers applet onto the overhead screen. She placed the expression $5 + 10 \times 2$ in the red pan and asked students to suggest an expression for the blue pan that would cause the scale to balance. Some students suggested 15×2, but the scale didn't balance; it showed that the blue pan was heavier.

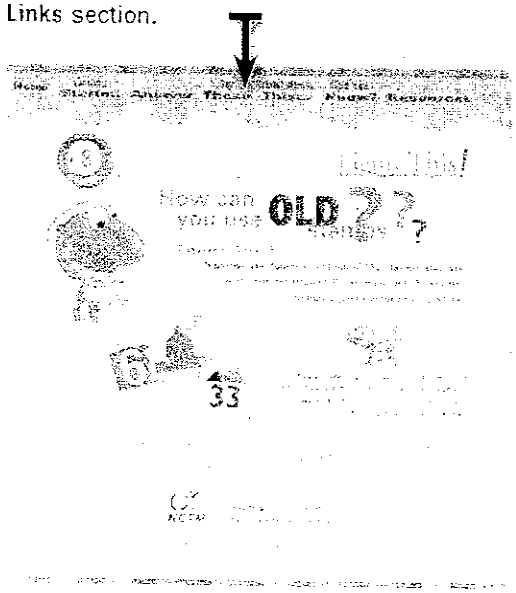


Teaching Scenario: Secondary *continued*

④ The next day, Ms. Naylor takes her class to the computer lab. She allows students to use the applet on their own to see what they can discover. She gives them a list of expressions to enter, and asks them to find equivalent expressions. Through this exploration, students realize that multiplication and division should be done before addition and subtraction. One student also realizes that multiplication and division must happen in order from left to right — for instance, the expression $4 \div 2 \times 3$ produces the correct answer of 6 if the division is performed first, but the incorrect answer 2/3 will be obtained if the multiplication is performed first.

As students work, Ms. Naylor noted, "The students didn't focus on the number above each pan. They didn't focus on the solution. They focused on *why*. This became a proof activity" for them, not just a computation exercise. The benefit of using the Illuminations tool, Ms. Naylor discovered, is that students are more willing to write out each step when simplifying complex expressions.

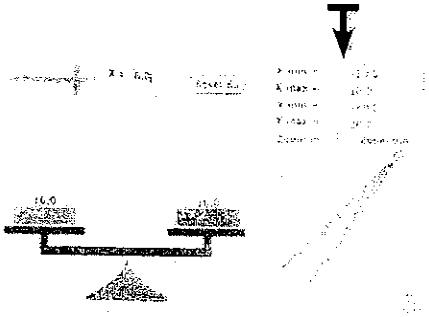
⑤ NCTM's *Principles and Standards for School Mathematics* suggests that students should develop conceptual understanding in addition to computational skills. Knowing this, Ms. Naylor wants to make sure that her students understand how the order of operations works, so she uses the **Old Stamps** problem from the Figure This! site, a resource from the National Council of Teachers of Mathematics that appears in the Illuminations' **Web Links** section.



The solution to the Old Stamps problem could be found with the expression $4 \times 33 + 3 \times 15$. Ms. Naylor asks her students:

- What is the value of this expression?
- What does this expression represent, in the context of the Old Stamps problem?
- How did you use the order of operations to determine the value of the expression?

"I love the activities on the Illuminations site! I teach in a small school with little money for software and technology, so this site is a huge help."

— Nichole Hoffman
7-12 Mathematics Teacher: Summit, S.D.

⑥ Later in the year, Ms. Naylor's algebra students will be learning to solve equations involving complex expressions. Knowing that Illuminations has over 60 interactive applets, she looks to see if there are others she can use with her class. She returns to the **Activity Search** page and searches for other grade 6-8 lessons on "algebra." In the list of results, she discovers the **Pan Balance – Expressions** activity. This applet is similar to **Pan Balance – Numbers**, so she is sure that students will have no trouble using it. However, instead of just entering numerical expressions, students can also enter expressions that involve variables. For instance, a student could enter $3 \times x - 2$ into the red pan and $2 \times x + 4$ into the blue pan; then, by adjusting the slider, the student could see that the pans are balanced and the graphs of the lines intersect when $x = 6$.